NAT@Logic 2015

LSFA 2015
GeTFun 3.0
FILOMENA 2
LFIs^15
TRS Reasoning School

Elaine Pimentel,
João Marcos,
Carlos Olarte,
Samir Gorsky,
Carolina Blasio,
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Sanderson Molick (Org.)

Book of Abstracts (1st Edition)
Natal-RN
UFRN
2015

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Book of Abstracts - NAT@Logic

Local organizers: Elaine Pimentel, João Marcos, Carlos Olarte, Carolina Blasio, Evelyn Erickson, João Daniel Dantas, Patrick Terrematte, Sanderson Molick.

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2. GeTFun 3.0
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Concept of design: Ulrike Harbort. LaTeX classes: Timm Lichte.

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Contents

Preface

As it is widely known, the beautiful city of Natal is probably the best place to do Logic in Brazil! From **Aug 31 to Sep 4, 2015**, it will be even more so, as we are preparing for you a fascinating programme for the **NAT@Logic 2015**, boasting a number of striking attractions, including 10 keynote speakers, 17 tutorials, and 67 contributed talks, distributed into several workshops related to **Logic** in *Computer Science*, in *Philosophy*, and in *Mathematics*.

Be sure to check the full detailed programme of **NAT@Logic 2015** here: http://tinyurl.com/NATALogic-slots.

August 2015.

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1 Keynote Speakers

Here are the **keynote speakers** of **NAT@Logic 2015**:

- Alessandra Palmigiano, TU Delft
- Arnon Avron, Tel Aviv University
- Dale Miller, INRIA Saclay & LIX
- Guillaume Hoffmann, CONICET - Universidad Blas Pascal
- Heinrich Wansing, Ruhr-Universität Bochum
- João Marcos, UFRN
- Luiz Carlos Pereira, PUC-Rio
- Ofer Arieli, The Academic College of Tel-Aviv
- Valentin Goranko, Stockholm University
- Valeria de Paiva, Nuance Communications

1.1 Unified Correspondence as a Proof-Theoretic Tool

Alessandra Palmigiano
TBM – Delft University of Technology
www.appliedlogictudelft.nl
(joint work with Giuseppe Greco, Minghui Ma, Apostolos Tzimoulis and Zhiguang Zhao)

Abstract. This presentation reports on the results of [Greco et al. 2015], and focuses on the formal connections between correspondence phenomena, well known from the area of modal logic, and the theory of display calculi, originated by [Belnap 1982].

Sahlqvist correspondence theory. Sahlqvist theory [Sahlqvist 1975] is among the most celebrated and useful results of the classical theory of modal logic, and one of the hallmarks of its success. It provides an algorithmic, syntactic identification of a class of modal formulas whose associated normal modal logics are *strongly complete* with respect to *elementary* (i.e. first-order definable) classes of frames.

Unified correspondence. In recent years, building on duality-theoretic insights [Conradie et al. 2014b], Sahlqvist theory has significantly broadened its scope, extending the benefits it originally imparted to modal logic to a wide range of logics which includes, among others, intuitionistic and distributive lattice-based (normal modal) logics [Conradie and Palmigiano 2012], non-normal (regular) modal logics [Palmigiano et al. 2015b], substructural logics [Conradie and Palmigiano 2015], hybrid logics [Conradie and Robinson 2015], and mu-calculus [Conradie and Craig 2015, Conradie et al. 2015].

The breadth of this work has stimulated many and varied applications. Some are closely related to the core concerns of the theory itself, such as the understanding of the relationship between different methodologies for obtaining canonicity results [Palmigiano et al. 2015a], or of the phenomenon of pseudocorrespondence [Conradie et al. 2014c]. Other, possibly surprising applications include the dual characterizations of classes of finite lattices [Frittella et al. 2015]. These and other results have given rise to a theory called *unified correspondence* [Conradie et al. 2014a].

Tools of unified correspondence theory. The most important technical tools in unified correspondence are: (a) a very general syntactic definition of the class of Sahlqvist formulas, which applies uniformly to each logical signature and is given purely in terms of the order-theoretic properties of the algebraic interpretations of the logical connectives; (b) the algorithm ALBA, which effectively computes first-order correspondents of input term-inequalities, and is guaranteed to succeed on a wide class of inequalities (the so-called *inductive* inequalities) which, like the Sahlqvist class, can be defined uniformly in each mentioned signature, and which properly and significantly extends the Sahlqvist class.

Unified correspondence and display calculi. The proposed talk focuses on an entirely different type of application of unified correspondence: the identification of the syntactic shape of axioms which can be translated into analytic rules of a display calculus. A rule is called *analytic* if adding it to a display calculus preserves Belnap's cut-elimination theorem. The connections between Sahlqvist theory and display calculi have been seminally observed by [Kracht 1996], in the context of his characterisation of those formulas of the language of basic modal logic (which he calls *primitive formulas*) which can be effectively transformed into structural rules of display calculi.

Contributions. The two tools of unified correspondence can be put to use to generalise Kracht's transformation procedure from axioms into analytic rules. This generalisation concerns more than one aspect. Firstly, in the same way in which the definitions of Sahlqvist and inductive inequalities can be given uniformly in each logical signature, the definition of primitive formulas/inequalities is introduced for any logical framework the algebraic semantics of which is based on distributive lattices with operators. Secondly, in the context of each such logical framework, we introduce a hierarchy of subclasses of inductive inequalities, progressively extending the primitive inequalities, the largest of which is the class of so-called *analytic inductive inequalities*. This class significantly generalises the class of primitive formulas/inequalities. We provide an effective procedure, based on ALBA, which transforms each analytic inductive inequality into an equivalent set of analytic rules.

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1.2 What is a classical LFI, and when are two such logics identical

Arnon Avron

School of Computer Science
Tel-Aviv University

Abstract. We discuss two fundamental questions concerning classical LFIs (i.e., LFIs which are based on positive classical logic). We start with the observation that according to the official definition, an axiomatic extension of classical logic is an LFI w.r.t some unary connective * iff it has a bottom element, and * satisfies three simple minimal conditions (that, for atomic P, neither of P and *P implies the other, and their conjunction does not implies the bottom element). This means that the class of classical LFIs is rather broad in comparison to the family of logics on which most of the investigations on LFIs have concentrated on. Accordingly, we provide a characterization of this family.

Another question that we address is when should we see two LFIs as identical. For example: is da Costa C_1 identical with the system $Cila$x, or Cla? Our reply is that with neither.

To provide adequate answers to both questions we describe and use appropriate semantic and proof-theoretical tools.

1.3 Defining the semantics of proof evidence

Dale Miller

Inria Saclay and Lix
École Polytechnique

Abstract. If automatic and interactive theorem provers store completed proofs, they do so using a range of proof structures, such as natural deduction, tableaux, resolution, and winning strategies. Ad hoc prover-specific proof scripts are also commonly used. I will outline how recent results on focused proof systems can be used to provide a formal framework for defining the meaning of a wide range of proof evidence. Interpreters of such formal definitions can thus be used as proof checkers. In order to make it possible to elide many details in formal proofs, proof checkers will be expected to perform significant proof reconstruction via deterministic and non-deterministic computations: such mixing of computation and deduction is an explicit feature of focused proof systems. I will discuss some of the ramifications of employing this framework on the ability of machines and humans to trust and communicate formal proofs.

1.4 Relation- and domain-changing modal operators

Guillaume Hoffmann

CONICET - Universidad Blas Pascal

Abstract. This talk presents two families of dynamic modal operators. We are interested in these operators because of their connection with hybrid logics and dynamic epistemic logics. We are driven by their semantic definitions and properties. In both cases the axiomatization of these logics is currently unknown.

First, we present a family of dynamic modal operators that can change the accessibility relation of a Kripke model during the evaluation of a modal formula (Relation-Changing, RC). In particular, these operators are able to delete, add or swap an edge between pairs of elements of the domain. We show these RC modal logics are fragments of classical logics and can also be translated to hybrid logics with binders. We also show that their satisfiability problems is undecidable.

Then, we present the dynamic modal operators Copy and Remove (C & R). The Copy operator replicates a given model, and the Remove operator removes paths in a given model. We show that the product update by an action model in dynamic epistemic logic decomposes in C & R operations. We also show that C & R operators with paths of length 1 can be expressed by action models with post-conditions. We investigate the expressive power of the logic with copy and remove operations, and prove decidability of the satisfiability problems of some of its syntactic fragments.

1.5 Compositionality in proof-theoretic semantics

Heinrich Wansing

Ruhr University Bochum

Abstract. In extensional many-valued logic, compositionality just means truth-functionality. The notion of truth is represented by a set of designated values, and semantical consequence (entailment) is defined as truth preservation from the premises of an inference to its conclusion. If falsity is understood as the absence of truth, the preservation of falsity is just the inverse of entailment viewed as truth preservation. The distinction between designated values (representing truth) and anti-designated values (representing falsity), however, has given rise to additional conceptions of entailment such as quasi-entailment and plausibility-entailment, and preservation of falsity is no longer the inverse of the preservation of truth.

The proof-theoretic counterpart of entailment is provability from a set of assumptions. In the talk I will argue that with truth and falsity as independent semantical categories, a compositional proof-theoretical account of the meaning of the logical operations requires a multiple-consequence setting, so that in addition to provability one has to employ a relation of dual provability or of disprovability. These considerations will be exemplified with an extended natural deduction framework.

1.6 On the birth of the LFIs: some alternative histories

João Marcos

Department of Informatics and Applied Mathematics
Federal University of Rio Grande do Norte

Abstract. The Logics of Formal Inconsistency (LFIs) were crafted with the intention of subsuming a large amount of logical systems developed in and around the Brazilian School of Paraconsistency, following the general intuition that the metatheoretical notion of *consistency* was to be internalized at the object language level. Some side projects that soon appeared related to distinguishing the notions of "contradiction" and "inconsistency", investigating the interaction between the paraconsistent negation and the consistency connective, studying the propagation of consistency, uncovering paraconsistent logics that constituted maximal fragments of classical logic, finding out which LFIs had natural and useful many-valued semantics, and which such systems had natural modal semantics.

The story could have been very different (or pretty similar) had we concentrated on other aspects of the logical systems that would give origin to the LFIs. We could have for instance decided to study logical systems containing a paraconsistent negation and a bottom particle, or study logics with more than one negation (at least one of them being paraconsistent), or study logics with different "levels of trivialization", or study logics that "dualized" constructive logics, and so on. But in fact we chose to concentrate on logics that allowed for the proof of some appropriate form of *derivability adjustment theorem*, whereby consistent reasoning would be recoverable. What was to be gained, and what has been lost? This talk will comment on some of the mentioned alternatives, and their interrelations.

1.7 Schematic rules and atomic polymorphism

Luiz Carlos Pereira

(PUC-Rio/UERJ)
(joint work with Edward Hermann Haeusler, PUC-Rio)

Abstract. What's an introduction rule for an operator ϕ? What's an elimination rule for ϕ? In 1978 Dag Prawitz proposed an answer to these questions by means of schematic introduction and elimination rules. Prawitz also proposed a constructive version of the well-known classical truth-functional completeness: if the introduction and elimination rules for an operator ϕ are instances of the schematic introduction and elimination rules, then ϕ is intutionistically definable. It can be shown that a weak-completeness result holds for a flat variant of Prawitz' schemata and that full completeness holds for the generalized schemata introduced by Peter Schröder-Heister. These schematic introduction and elimination rules can also be used to show that logics whose operators satisfy the schematic rules and whose derivations satisfy the sub-formula principle can be translate into minimal implicational logic. In 2013 Fernando and Gilda Ferreira introduced the system for atomic polymorphism. This system can be characterized as a second order propositional logic in the language $\forall$, $\rightarrow$ such that $\forall$-elimination is restricted to atomic instantiations. The aim of the present paper is twofold: (1) to use atomic polymorphism to study the proof theory of schematic systems and (2) to produce high-level translations for a large class of logics.

1.8 Sequent-based Argumentation

Ofer Arieli
School of Computer Science
The Academic College of Tel-Aviv
(joint work with Christian Strasser – U. Bochum)

Abstract. Logic-based approaches for analyzing and evaluating arguments have been largely studied in recent years, yielding a variety of formal methods for argumentation-based reasoning. In this talk we introduce an abstract, proof theoretical approach to logical argumentation. This is realized in three aspects:

- *Arguments are represented by sequents.* Sequents can be regarded as specific kinds of judgments and so they are useful for representing logical arguments. This allows us to incorporate sequent-based approaches in argumentation theory, like using sequent calculi for producing arguments in an automated way. Moreover, some restrictions in previous definitions of logical arguments, such as minimality and consistency of support sets are lifted, allowing for a more flexible way of expressing arguments.
- *Conflicts between arguments are represented by sequent elimination rules.* Interactions between arguments (expressed by attack relations) are represented in terms of Gentzen-style rules of inference. This induces a general and uniform approach not only for introducing arguments, but also for eliminating them.
- *Deductions are made by dynamic proof systems.* For explicating actual reasoning in an argumentation framework we extend the standard notion of proofs in sequent calculi. Generally, the fact that an argument can be challenged (and possibly withdrawn) by a counter-argument is reflected in dynamic proofs by the ability to consider certain formulas as not derived at a certain stage of the proof even if they were considered derived in earlier stages of the proof. We show how despite of this non-monotonic nature of dynamic derivations one may still draw irreversible conclusions by using them.

Keeping our sequent-based setting generic and modular allows us to accommodate different types of languages and logics, including non-classical ones. We demonstrate the usefulness of our approach by means of various examples, and show that this approach is rich enough to capture a variety of paradigms for handling conflicting arguments.

1.9 A logical framework for multi-agent visual-epistemic reasoning

Valentin Goranko

Department of Philosophy
Stockholm University
(joint work with Olivier Gasquet and François Schwarzentruber)

Abstract. We study a logical framework for multi-agent epistemic reasoning based on processing of visual information. This framework is modelled by multi-agent systems where each agent receives visual information from the environment via mobile camera with a given angle of vision in the plane. The agents can thus observe their surroundings and each other and can reason about each other's observation abilities and knowledge derived from these observations. We introduce suitable logical languages for describing such scenarios and formalising such reasoning, involving atomic formulae stating what agents can see, multi-agent epistemic operators for individual, distributed and common knowledge, as well as dynamic operators reflecting the ability of agents (or, their cameras) to move and turn around in order to reach positions satisfying formally specified visual-epistemic requirements. We introduce and study different versions of the semantics for these languages and develop algorithmic methods for automated reasoning in our basic logical system, called 'Big Brother Logic', and some natural extensions of it.

In this talk I will present the logical framework of BBL and will discuss the interaction between observational abilities and knowledge, both of which essentially depend on the underlying geometric constraints and assumptions. Besides being of purely logical interest, this work has potential applications to formal specification, verification and automated reasoning in multi-robot systems.

1.10 Modal type theory

Valeria de Paiva

Nuance Communication
(joint work with Eike Ritter)

Abstract. Type theory does not usually discuss logical modalities, and modalities tend to be mostly studied within classical logic, not type theory. But modalities should be useful in type theory, as they are generally very useful in theoretical computer science.

Since there seems to be a renewed interest in the notions of constructive modal type theory and linear type theory, in part caused by the interest in homotopy type theory, it seems sensible to recapitulate some of the known facts, especially ones on the semantics of modal type theory. I will describe a fibrational categorical semantics for the necessity-only fragment of constructive modal type theory, both with and without dependent types.

Dependent type theories are usually, but not always, given categorical semantics in terms of fibrations. We provide semantics in terms of fibrations for both the non-dependent and the dependent modal type systems proposed and prove them sound and complete. We also discuss some of the possible alternatives.

2 10th LSFA: Logical and Semantic Frameworks, with Applications

Dates: Aug 31-Sep 1, 2015

Logical and semantic frameworks are formal languages used to represent logics, languages and systems. These frameworks provide foundations for formal specification of systems and programming languages, supporting tool development and reasoning.

The **objective** of this workshop is to bring together theoreticians and practitioners from the field of Computational Logic to promote new techniques and results, and to facilitate feedback on the implementation and application of such techniques and results in practice.

Topics of interest to this forum include, but are not limited to:

- Logical frameworks
- Proof theory
- Type theory
- Automated deduction
- Semantic frameworks
- Specification languages and meta-languages
- Formal semantics of languages and systems
- Computational and logical properties of semantic frameworks
- Implementation of logical or semantic frameworks
- Applications of logical or semantic frameworks

LSFA 2015 also aims to be a forum for presenting and discussing work in progress, and therefore to provide feedback to authors on their preliminary research. The final proceedings are produced immediately after the meeting, so that authors can incorporate the feedback in the published papers.

The **invited speakers** this year include:

- Ofer Arieli, The Academic College of Tel-Aviv
  Talk: "Sequent-based Argumentation"
- Valentin Goranko, Stockholm University
  Talk: "A logical framework for multi-agent visual-epistemic reasoning"

- Dale Miller, INRIA Saclay & LIX
  Talk: "Defining the semantics of proof evidence"
- Valeria de Paiva, Nuance Communications
  Talk: "Modal type theory"

For more information on LSFA 2015, please follow this link:
https://www.mat.ufrn.br/~LSFA2015/

Previous LSFAs are described here:
http://lsfa.cic.unb.br/

2.1 Type Soundness for Path Polymorphism

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Abstract. *Path polymorphism* is the ability to define functions that can operate uniformly over arbitrary recursively specified data structures. Its essence is captured by patterns of the form $x\,y$ which decompose a compound into its parts. Typing these kinds of patterns is challenging since, from the classical approach, the type of the compound does not determine the type of its components. We propose a static type system (*i.e.* no run-time analysis) for a pattern calculus that captures this feature. Our solution combines type application, constants as types, union types and recursive types. We address the fundamental properties of subject reduction and progress that guarantee a well-behaved dynamics. Both these results rely crucially on a notion of *pattern compatibility* and also on a coinductive characterisation of subtyping.

2.2 Completeness in PVS of a Nominal Unification Algorithm

Mauricio Ayala-Rincón[1], Maribel Fernández, and Ana Cristina Rocha-Oliveira

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Abstract. Nominal systems are an alternative approach for the treatment of variables in computational systems. In the nominal approach variable bindings are represented using techniques that are close to first-order logical techniques, instead of using a higher-order metalanguage. Functional nominal computation can be modelled through *nominal rewriting*, in which α-equivalence, nominal matching and nominal unification play an important role. Nominal unification was initially studied by Urban, Pitts and Gabbay and then formalised by Urban in the proof assistant Isabelle/HOL and by Kumar and Norrish in HOL4. In this work, we present a new specification of nominal unification in the language of PVS and a formalisation of its completeness. This formalisation is based on a natural notion of nominal α-equivalence, avoiding in this way the use of the intermediate auxiliary weak α-relation considered in previous formalisations. Also, in our specification, instead of applying simplification rules to unification and freshness constraints, we recursively build solutions for the original problem through a straightforward functional specification, obtaining a formalisation that is closer to algorithmic implementations. This is possible by the independence of freshness contexts guaranteed by a series of technical lemmas.

2.3 Strong Normalization through Intersection Types and Memory

Antonio Bucciarelli[1], Delia Kesner[1], and Daniel Ventura[2]

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Abstract. We characterize β-strongly normalizing λ-terms by means of a non-idempotent intersection type system. More precisely, we first define a memory calculus $\mathtt{K}$ together with a non-idempotent intersection type system $\mathcal{K}$, and we show that a $\mathtt{K}$-term t is typable in $\mathcal{K}$ if and only if t is $\mathtt{K}$-strongly normalizing. We then show that β-strong normalization is equivalent to $\mathtt{K}$-strong normalization. We conclude since λ-terms are strictly included in $\mathtt{K}$-terms.

2.4 Principles of Alpha-Structural Induction and Recursion for the Lambda Calculus in Constructive Type Theory

Ernesto Copello[1], Álvaro Tasistro[1], Nora Szasz[1], Ana Bove[2], and Maribel Fernández[3]

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Abstract. We formulate principles of induction and recursion for a variant of lambda calculus in its original syntax (i.e., with only one sort of names) where α-conversion is based upon name swapping as in nominal abstract syntax. The principles allow to work modulo α-conversion and implement the Barendregt variable convention. We derive them all from the simple structural induction principle on concrete terms and work out applications to some fundamental meta-theoretical results, such as the substitution lemma for α-conversion and the lemma on substitution composition. The whole work is implemented in Agda.

2.5 Multi-focused Proofs with Different Polarity Assignments

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Brazil

Abstract. In this work, we will reason on how a given focused proof where atoms are assigned with some polarity can be transformed into another focused proof where the polarity assignment to atoms is changed. This will allow, in principle, transforming a proof obtained using one proof system into a proof using another proof system. More specifically, using the intuitionistic focused system LJF restricted to Harrop formulas, we define a procedure, introducing cuts, for transforming a focused proof where an atom is assigned with positive polarity into another focused proof where the same atom is assigned negative polarity and vice-versa. Then we show how to eliminate these cuts, obtaining a very interesting result: while the process of eliminating a cut on a positive atom gives rise to a proof with one smaller cut, in the negative case the number of introduced cuts grows exponentially. We end the paper by showing how to use maximal multi-focusing identify proofs in LJF, giving rise to a 1-1 translation between maximal proofs in LJF and proofs in the natural deduction system for intuitionistic logic NJ, restricted to Harrop formulas.

2.6 On Strong Normalization in Proof-graphs for Propositional Logic

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Abstract. Traditional proof theory of Propositional Logic deals with proofs whose size can be huge. Proof theoretical studies discovered exponential gaps between normal or cut free proofs and their respective non-normal proofs. The use of proof-graphs, instead of trees or lists, for representing proofs is getting popular among proof-theoreticians. Proof-graphs serve as a way to provide a better symmetry to the semantics of proofs and a way to study complexity of propositional proofs and to provide more efficient theorem provers, concerning size of propositional proofs.

Mimp-graphs were initially developed for minimal implicational logic representing proofs through references rather than copy. Thus, formulas and sub-deductions preserved in the graph structure, can be shared deleting unnecessary sub-deductions resulting in the reduced proof. In this work, we consider full minimal propositional logic and show how to reduce (eliminating maximal formulas) these representations such that strong normalization theorem can be proved by simply counting the number of maximal formulas in the original derivation. In proof-graphs, the main reason for obtaining the strong normalization property using such simple complexity measure is a direct consequence of the fact that each formula occurs only once in the proof-graph and the case of the hidden maximum formula that usually occurs in the tree-form derivation is already represented in the mimp-graph.

2.7 Normalization of N-Graphs via Sub-N-Graphs

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Abstract. Alves presented in his PhD thesis a normalization procedure for N-Graphs, a multiple conclusion natural deduction for propositional classical logic proposed by de Oliveira in 2001, with proofs as directed graphs. Here we develop a new normalization for N-Graphs inspired by A. Carbone's work in 1999, where she proposed a combinatorial model to study the evolution of proofs during the procedure of cut elimination.

2.8 On Graphs for Intuitionistic Modal Logics

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Abstract. We present a graph approach to intuitionistic modal logics, which provides uniform formalisms for expressing, analysing and comparing Kripke-like semantics. This approach uses the flexibility of graph calculi to express directly and intuitively possible-world semantics for intuitionistic modal logics. We illustrate the benefits of these ideas by applying them to some familiar cases of intuitionistic multi-modal semantics.

2.9 Classical resolution for many-valued logics

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Abstract. We present a resolution-based proof method for finite-valued propositional logics based on an algorithmic reduction procedure that expresses these logics in terms of bivalent semantics. Our approach is hybrid in using some elements which are internal and others which are external to the many-valued logic under consideration, as we embed its original language into a more expressive metalanguage to deal with the satisfiability problem. In contrast to previous approaches to the same problem, our target language is fully classical, what turns the design of the resolution-based rules for a specific many-valued logic into a straightforward task. Correctness results, which are proved in detail in the present study, follow easily from results on classical resolution. Implementation of reasoning tools can be achieved by direct translation into classical propositional logic and making use of reliable existing automated provers. We illustrate the application of the method with examples.

2.10 Computational paths, identity type, and the groupoid model

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Abstract. We introduce a new way of formalizing the intensional identity type based on the notion of computational paths which will be taken to be proofs of propositional equality, and thus terms of the identity type. Our approach results in an elimination rule which is much simpler and much more usable than the one given by Martin-Löf in his intensional identity type. In order to make this point clear we construct terms of some relevant types using our proposed elimination rule. We also show that one of the properties of Martin-Löf's original identity type is present on our formulation of the identity type of computational paths. We are referring to the fact that the identity type induces a groupoid structure, as proposed by Hofmann & Streicher (1994). Using categorical semantics, we show that computational paths induce a groupoid structure too. It is further shown that computational paths induce higher categorical structures.

2.11 Formalization of simplification for context-free grammars

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Abstract. Context-free grammar simplification is a subject of high importance in computer language processing technology as well as in formal language theory. This paper presents a formalization, using the Coq proof assistant, of the fact that general context-free grammars generate languages that can be also generated by simpler and equivalent context-free grammars. Namely, useless symbol elimination, inaccessible symbol elimination, unit rules elimination and empty rules elimination operations were described and proven correct with respect to the preservation of the language generated by the original grammar.

2.12 Nondeterministic Linear Automata and a Class of Deterministic Linear Languages

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Abstract. In this paper we consider the class of λ-nondeterministic linear automata as a model of the class of linear languages. As usual in other automata models, λ-moves do not increases the acceptance power. The main contribution of this paper is to introduce the deterministic linear automata and even linear automata, i.e. the natural restriction of nondeterministic linear automata for the deterministic and even linear language classes. In particular, there are different not equivalent proposed for the class of "deterministic" linear languages and here we proved that the class of languages accepted by the proposed deterministic linear automata are not contained in any of the these classes and in fact contain properly the most of these classes.

2.13 Checking Overlaps of Nominal Rewriting Rules

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Abstract. Nominal rewriting generalises first-order rewriting by providing support for the specification of binding operators. In this paper, we give sufficient conditions for (local) confluence of closed nominal rewriting theories, based on the analysis of rule overlaps. More precisely, we show that closed nominal rewriting rules where all proper critical pairs are joinable are locally confluent. We also show how to refine the notion of rule overlap to derive confluence of the closed rewriting relation. The conditions that we define are easy to check using a nominal matching algorithm.

2.14 Proving Correctness of a Compiler Using Step-indexed Logical Relations

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Abstract. In this paper we prove the correctness of a compiler for a call-by-name language using step-indexed logical relations and biorthogonality. The source language is an extension of the simply typed lambda-calculus with recursion, and the target language is an extension of the Krivine abstract machine. We formalized the proof in the Coq proof assistant.

2.15 Canonical HybridLF: Extending Hybrid with Dependent Types

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Abstract. We introduce Canonical HybridLF (CHLF), a metalogic for proving properties of deductive systems, implemented in Isabelle HOL. CHLF is closely related to two other metalogics. The first is the Edinburgh Logical Framework (LF) by Harper, Honsell and Plotkin. The second is the Isabelle HOL Hybrid system developed by Ambler, Crole and Momigliano which supports use of Higher-Order Abstract Syntax (HOAS) through an un-typed lambda calculus.

Historically there are two problems with HOAS: its incompatibility with inductive types and the presence of exotic terms. Hybrid provides a partial solution to these problems whereby HOAS functions that include metalogic bound variables are automatically converted to a machine-friendly de Bruijn representation hidden from the user.

The key innovation of CHLF is the replacement of the un-typed lambda calculus with a dependently-typed lambda calculus in the style of LF. CHLF allows signatures containing constants representing the judgements and syntax of an object logic, together with proofs of metatheorems about its judgements, to be entered using a HOAS interface. Proofs that metatheorems defined in the signature are valid are created using the M2 metalogic of Schurmann and Pfenning.

We make a number of advances over existing versions of Hybrid: we now have the utility of dependent types; the unitary bound variable capability of Hybrid is now potentially finitary; and the old method of indicating errors using special elements of core datatypes is replaced with a more streamlined one that uses the Isabelle option type.

CHLF has an advantage over Twelf in that a proof that a metatheorem holds is explicitly stated, rather than being automatically generated and hidden from the user. This brings difficulties, however, as the search for a proof can be arduous when executed by hand. It is future work to develop domain specific automatic search procedures.

3 3rd GeTFun: Generalizations of Truth-Functionality

Dates: Sep 1-3, 2015.

This will be the third workshop of the GeTFun project – http://sqig.math.ist.utl.pt/GeTFun/ – , on generalizations of truth-functionality, investigated from the viewpoints of Universal Logic, Proof Theory and Formal Semantics. We are looking for advances which are at once philosophically well-motivated, mathematically well-developed, and computationally well-behaved.

Keynote speakers, this year, include:

- Alessandra Palmigiano, TU Delft
  Talk: "Unified Correspondence as a Proof-Theoretic Tool"
- Guillaume Hoffmann, CONICET - Universidad Blas Pascal
  Talk: "Relation- and Domain-Changing modal operators
- Luiz Carlos Pereira, PUC-Rio
  Talk: "Schematic rules and atomic polymorphism"
- Heinrich Wansing, Ruhr University Bochum
  Talk: "Compositionality in proof-theoretic semantics"

For more information on GeTFun 3.0, please follow this link. (http://sqig.math.ist.utl.pt/GeTFun/3.0).

3.1 Agreeing about Disagreement

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Abstract. An essential distinction is made between the concepts of information and belief. Then four different logics of belief result from a four-valued system, each of these corresponding to specific truth-functional constraints upon truth- and falsity-claims. After exploring their seemingly modal properties, it is argued that a comparison of these belief operators helps to characterize various relations of agreement and disagreement between speakers.

A Logic for Information. We propose a general Logic of Acceptance and Rejection, $\mathbf{AR}_4$. This consists of a set of formulas with four sentential connectives: negation $\neg$, conjunction $\wedge$, disjunction $\vee$, and conditional $\rightarrow$. Let $\mathbf{A}(p)$ the value assigned to a sentence p by an arbitrary speaker, where 1 stands for yes-answers (acceptance) and 0 for no-answers (rejection). The independence of truth and falsity leads to a double valuation $\mathbf{A}(p) = \langle \mathbf{a}_1(p), \mathbf{a}_2(p) \rangle$ about p's truth and p's falsity, respectively. This framework results in the following matrices, where the striking definition of conditional makes a crucial difference between Belnap's system FDE [Belnap 1982] and $\mathbf{AR}_4$ [Schang and Trafford 2015]:

A(p)	A(¬p)
11	11
10	01
01	10
00	00

A(p ∧ q)

	11	10	01	00
11	11	11	01	01
10	11	10	01	00
01	01	01	01	01
00	01	00	01	00

A(p ∨ q)

	11	10	01	00
11	11	10	11	10
10	10	10	10	10
01	11	10	01	00
00	10	10	00	00

A(p → q)

	11	10	01	00
11	10	10	01	00
10	10	10	01	00
01	01	00	00	00
00	00	00	00	00

An objection to such a logic is that it makes a confusion between truth-values (truth, falsity) and propositional attitudes (belief, disbelief), thereby leading to counterintuitive theorems [Dubois 2008]. A reply is to say that logical values are not beliefs but states of information, that is, evidence for or against the truth and falsity of sentences [Wansing and Belnap 2010]. In the light of this *epistemic* definition of truth-values, we can devise $\mathbf{AR}_4$ as a neutral background for competing logics of disagreement.

A Logic for Belief. Now how can any two speakers understand each other if they do not share the same criteria for agreement and disagreement? This can be explained by an extension of $\mathbf{AR}_4$ into $\mathbf{AR}_{4_\blacksquare} = \mathbf{AR}_4 + \{\blacksquare, \sim\}$, where $\blacksquare$ is a set of various unary operators of belief and $\sim$ the symbol of *epistemic* (outward) negation for complete disagreement (by contradistinction to the *ontic*, inward negation $\neg$). Taking the case of any sentence p, each of the four aforementioned believers can be described by a specific belief operator with various constraints about p's truth.

(a') Optimists favor p's truth over its falsity and accept p so long as no conclusive argument is given against p.
For every $\mathbf{A}(p)$, $\mathbf{A}$(◫ p) = 1 iff $\mathbf{a}_1(p) = 1$ or $\mathbf{a}_2(p) = 0$.

(b') Pessimists favor p's falsity over its truth and accept $\neg$ p so long as no conclusive argument is given for p.
For every $\mathbf{A}(p)$, $\mathbf{A}(\boxminus p) = 1$ iff $\mathbf{a}_1(p) = 0$ or $\mathbf{a}_2(p) = 1$.

(c') Skeptics believe what for which there is a conclusive argument for and no conclusive against.
For every $\mathbf{A}(p)$, $\mathbf{A}(\boxtimes p) = 1$ iff $\mathbf{a}_1(p) = 1$ and $\mathbf{a}_2(p) = 0$.

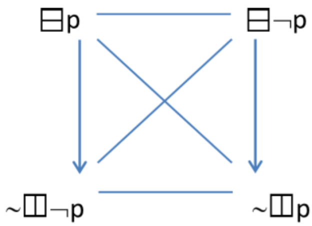
(d') Eclectics believe what there is an evidence for.
For every $\mathbf{A}(p)$, $\mathbf{A}(\boxplus p) = 1$ iff $\mathbf{a}_1(p) = 1$ or $\mathbf{a}_2(p) = 0$.

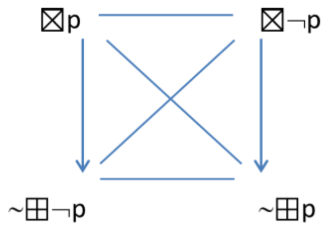
The difference in logics can be shown in the below matrix of belief operators:

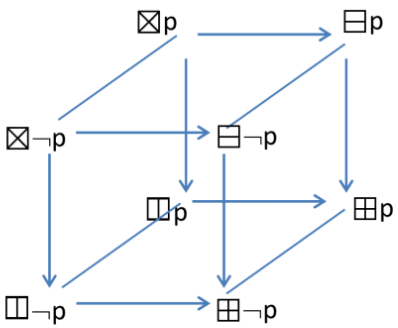
p	◫p	⊟p	⊠p	⊞p
11	10	01	00	11
10	10	10	10	10
01	01	01	01	01
00	10	01	00	11

After examining the main properties of these unary functions, we will see how they differ from modalities with respect to some general theorems like D-consistency, or S4- and S5-iterations. At the same time, these properties help to express a number of connections between the various logics of justification.

Logic, Consequence, and Opposition. This will lead to the final results concerning two main features of logic: *consequence*, and *opposition*. On the one hand, the total relations of *opposition* between believers can be displayed in two independent logical squares including *epistemic* negation, such that







On the other hand, four main logical relations result from our discussion about total and partial agreement and disagreement:

1. Values are preserved from premises to conclusion: *consequence* obtains with respect to truth (truth-preservation) or falsity (falsity-preservation) [Frankowski 2004], [Malinowski 1990].
2. Values are not preserved from thesis to objection: *opposition* obtains from two possible perspectives (truth-not-preservation) or falsity (falsity-not-preservation).
3. Rejectivism justifies the logical difference between not-X preservation and X not-preservation (with X for T or F) [Skura 2015].

The logical relations between the different believers are finally displayed in the following squares of total *opposition*, together with a more comprehensive cube relating total and partial *opposition*. These relations will be accounted according to the ordered values (a)(b)(c)(d) of the formulas, where each letter refers to a row of a given matrix.

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3.2 The Calculus of Natural Calculations

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Abstract. Gentzen introduces in his famous paper "Untersuchungen über das logische Schließen" two different calculi for formal syntactical reasoning. Before defining his sequent calculus, which is a technically convenient tool for proving his Hauptsatz, Gentzen discusses first the so called Calculus of Natural Deduction. The main advantage of the Calculus of Natural Deduction and the reason of its introduction is its close relationship to informal mathematical reasoning. In Gentzen's own words:[1]

> We wish to set up a formalism that reflects as accurately as possible the actual logical reasoning involved in mathematical proofs. (p. 74)
>
> The calculus lends itself in particular to the formalization of mathematical proofs. (p. 80)

Such calculi close to informal reasoning are philosophically interesting. They minimize the inevitable gap between an informal argumentation and its formalization; this minimalisation may justify to carry over formal results gained through an investigations of derivations to their informal counterparts. This way we are able to provide formally justified answers to philosophical questions about the properties of (informal) proofs. Furthermore, such calculi shed a light on the notion of "natural argumentation" itself and provides thereby good arguments to a philosophical discussion about the nature of reasoning.

With respect to argumentations involving only statements, Gentzen's Calculus of Natural Deduction is, in deed, pretty close to the argumentation schemata found in mathematics. But mathematicians do not only argue with statements; they also calculate within their proofs. The usual way to handle this phenomenon in a formal calculus is to introduce axioms or rules for equality statements and to translate the calculations into argumentations about equality statements. This is logically perfect, but not very natural.

In our talk, we present an alternative and natural approach to the formalisation of informal calculations: we extend the Calculus of Natural Deduction by adding some term inference rules (elimination and introduction rules for

[1] Quotes taken from: *The Collected Papers of Gerhard Gentzen*, M. E. Szabo (Editor), North Holland Publishing Company, 1969

equality statements), which allow to transfer the formal argumentation to a subatomic level, where we may calculate directly with terms before establishing an equality statement. Having introduced this Calculus of Natural Calculations, we provide some basic proof theoretic results about this calculus, in particular, we sketch its completeness and discuss briefly the problem of normalization.

This extension of Natural Deduction motivates further extensions of the calculus allowing the direct formalisation of other calculation schemata related to other (binary) relations found in mathematics as, for example, the "smaller-than" relation in arithmetic. We provide a brief overview about this possibilities.

Finally, we consider an extension of the calculus into another direction: we may introduce rules allowing to introduce end eliminate meta statements about formulae as, for example, statements expressing logical equivalence or consequence between formulae. Such rules allow the integration of sequent calculi into the Calculus of Natural Deduction and provide a uniform formal frame for the natural formalisation of mathematical reasoning in all of its relevant aspects.

3.3 Bounded-Degree Quantification and MSO_2

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Descriptive Complexity studies the expressive power of logical languages in terms of complexity classes. Each sentence of a given logical language is regarded as defining a decision problem on finite mathematical structures, and, hence, the entire logic defines a class of decision problems. It follows that the expressive power of a logic can be related to some complexity class, namely the complexity class to which the problems expressed by its formulas belong. The paradigmatic result of the area, and one of the first, is Fagin's Theorem, which states that the existential fragment of second-order logic (∃SO) captures the NP class, the class of problems solvable in polynomial non-deterministic time. It means that each sentence of the ∃SO defines a decision problem in NP and for each problem in NP there is a ∃SO sentence whose models are exactly the accepted instances of the problem. The result was extended to the entire Polynomial Hierarchy (PH), the problems solved by alternating Turing machines in polynomial time and constant number of alternations, which is captured by second-order logic as shown in Stockmeyer [1976].

Many results establishing the connection of logical languages and its fragments to complexity classes have been discovered (see Immerman [1999]). A well studied fragment of SO is the Monadic Second-Order logic (MSO). Büchi [1960] showed that MSO expresses, on the class of strings, exactly the regular languages. The result can be strengthened to show that, on strings, MSO has the same expressive power as its existential fragment ∃MSO, and, hence, each formula of ∃MSO defines a regular language and vice-versa. ∃MSO can define NP-complete problems on the class of graphs, for instance 3-colourability of graphs can be defined in ∃MSO. When we compare this fact with Büchi's Theorem, we see that, if we restrict the kind of structures on which the formulas are going to be evaluated, we can sometimes obtain a better upper-bound on the computational effort required to solve the problem defined by some logic. MSO has also gained attention due to Courcelle's Theorem (see Courcelle [1990]), which states that problems defined in MSO can be solved in linear time on graphs of bounded-tree width. Courcelle's result also holds for MSO_2, which is a variant of MSO on graphs where not only sets of vertices but also sets of edges can be quantified.

We are interested in a fragment of SO, namely the BDSO fragment, where quantification is restricted to relations of bounded-degree. The degree of a vertex is the number of its neighbours, and the degree of a graph is the maximum of the degrees of their vertices. The degree of a relation is the degree of the corresponding Gaifman graph, which is the graph whose vertices are those which appear in some tuple of the relation and whose edges connect vertices which appear in the same tuple. The existential fragment of BDSO has been studied by Grandjean and Olive [1998] and Durant et al. [1998], which related it to the class NLIN of problems solvable in linear time in a non-deterministic random access registers machine (NRAM). This result can be extended to show that BDSO captures alternating linear time on alternating random access machines with constant number of alternations (see Ferreira [2012]).

We want to establish the relation between BDSO and MSO_2 with respect to their expressive power. We can show that BDSO is more expressive then MSO_2 on some classes of structures. It is open whether there is a class of structures on which MSO_2 is more expressive than BDSO.

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3.4 Yoneda's embedding and Post completeness

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(joint work with Edward Hermann Haeusler)

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Abstract. We show that Hilbert's proof of Post completeness for the case of classical propositional logic is an instance of the Yoneda lemma, a meta-mathematical result in the field of category theory, thus allowing a proof in a categorical setting. At last we also discuss an interpretation of the lemma which prescribes a theorem logic tautologies satisfy. Such theorem is the one that is inconsistent with the assumption that classical logic could be extended, that is the core of Hilbert's proof, thus making the lemma doubly useful in our study.

3.5 Dialectica Categories, Cardinalities of the Continuum and Combinatorics of Ideals

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Abstract. We propose to revisit some old work of Andreas Blass connecting Dialectica categories, considered as models of Linear Logic, to tools of Vojtas that relate cardinal invariants via inequalities. We sharpen these results in the light of new developments in Set Theory, such as those introduced by Moore, Hrusak, and Dzamonja (parametrized Diamond principles) and described in Rangel's master thesis, written under the supervision of the first author. As a study case, we investigate how certain cardinals defined in terms of ideals (see, e.g. Hrusak's combinatorics of filters and ideals) are naturally encompassed by this approach — at least in the specific situation where those ideals are defined in terms of pre-orders (i.e., reflexive, transitive binary relations).

3.6 Analyticity and conservativity in the display calculus

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(joint work with Agata Ciabattoni[1] and Nikolaos Galatos[2])
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[2] University of Denver

Abstract. We consider how to obtain analytic display calculi for axiomatic extensions of a logic (using substructural logics as an example), and then discuss how the issue of conservativity between the logic of the display calculus and certain sublogics can be investigated directly in this setting.

In order to study a logic from a proof-theoretical perspective, it is necessary to first obtain an analytic proof-calculus for the logic. By an analytic proof-calculus we mean that the calculus has the subformula property i.e. any proof in the calculus (of some formula ψ, say) uses only those formulae that are subformulae of ψ. The point is that the proofs then have nice structural properties, facilitating further analysis. Despite much work on this topic since Gentzen's [Gentzen 1969] seminal work on analytic sequent calculi for intuitionistic and classical first-order logics, many logics still do not have an analytic proof-calculus. Even when an analytic calculus for the logic L is known, it is often unclear how to obtain an analytic calculus for an axiomatic extension L+Ax. The situation is particularly vexing since many logics are constructed as axiomatic extensions of existing logics.

Belnap's Display Calculus [Belnap 1982] is a proof-theoretic formalism generalising Gentzen's sequent calculus that is suitable for presenting logics whose logical operators are residuated. Indeed, the display calculus can be viewed as the proof-theoretic face of the algebraic semantics of a fully residuated logic. In particular, the residuation of the logical operators corresponds to a powerful structural property from the proof-theoretic perspective: the display property. Another attractive proof-theoretic feature is the general cut-elimination theorem which applies whenever the rules of the display calculus obey certain easy-to-verify conditions. Indeed, the formalism has been applied to give analytic calculi for many different families of logics including substructural logics, tense logics and bunched implication logics.

We first present a modular approach for computing analytic display calculi for axiomatic extensions of a logic. In particular, we identify a class of axioms such that every axiomatic extension using these axioms has an analytic calculi. Previous work on this topic has focussed on a concrete calculus for a logic—the display calculus [Kracht 1996] for tense logic and the hypersequent calculus [Ciabattoni et al. 2008] for Full Lambek logic. In contrast, our sufficient conditions are stated abstractly rather than for concrete calculi. Consequently, our results [Ciabattoni and Ramanayake 2013; 2014] apply to well-known display calculi, immediately yielding analytic calculi for axiomatic extensions of these logics. The set of axioms that we treat with our procedure is determined by the invertible rules of the base calculus. We can also show that it is decidable if an axiom belongs to this set or not.

Next we consider the reverse direction. We show that every structural rule satisfying the conditions for cut-elimination is equivalent to an axiom from the set we identified before. In this way we give a full characterisation of this set of structural rules. Kracht [Kracht 1996] has shown a similar characterisation to ours for the concrete case of tense logic. Kracht's result can be obtained as a special case of the result presented here.

We conclude by discussing the issue of conservativity that arises naturally when we wish to consider sublogics in a restricted language. Specifically, suppose that the logic L is defined in the language $\mathcal{L}$, and the logic $L' \subset L$ is defined in the (restricted) language $\mathcal{L}'$. Then L is said to be *conservative* over L' if every theorem of L in the restricted language $\mathcal{L}'$ is a theorem of the sublogic L'. Suppose that $\mathcal{C}$ is an analytic display calculus for L and suppose that δ is a derivation in $\mathcal{C}$ of a formula ψ in the language $\mathcal{L}'$. Now δ can be seen viewed in the usual way as a directed tree with root ψ (the directed edges correspond to the rules of the calculus and the nodes correspond to sequents). Notice that δ does *not* witness a proof of ψ in the sublogic L' because certain nodes in δ may not be even interpretable in $\mathcal{L}'$. However, if we can extract a new tree from δ whose nodes *are* interpretable in $\mathcal{L}'$, then conservativity amounts to showing that each edge in the new tree corresponds to a valid inference in L'. The point is that the interesting theoretical result of conservativity can be expressed proof-theoretically as a transformation on δ. Moreover, the conservativity result then yields an analytic calculus for L'. Conservativity [Clouston et al. 2013] of bi-intuitionistic linear logic over full-intuitionistic linear logic has already been shown in this this way. The analytic display calculi obtained above pave the way for the study of conservativity for a large class of logics.

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3.7 Grafting Hypersequents onto Nested Sequents

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(joint work with Roman Kuznets)

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Abstract. We introduce a new Gentzen-style framework of grafted hypersequents that combines the formalism of nested sequents with that of hypersequents. To illustrate the potential of the framework, we present novel Gentzen-style calculi for the modal logics K5 and KD5, as well as for extensions of the modal logics K and KD with the axiom for shift reflexivity. The latter of these extensions is also known as SDL+ in the context of deontic logic. All our calculi enjoy syntactic cut elimination and can be used in backwards proof search procedures of optimal complexity.

3.8 On proofs, systems, specification and verification

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Abstract. In a series of works, Ciabattoni et al. developed a systematic way of transforming axioms into sequent rules for a number of logics and proof systems. It can be shown, for example, that any extension of FL_{ew} (Full Lambek calculus with weakening and exchange) with axioms in the class $\mathcal{N}_2$ defines structural sequent rules corresponding to the additional axioms. Parallel to this work, Pimentel et al. used linear logic in order to encode various logics and reason about them. In this work, we show how to generate automatically clauses in linear logic with subsexponentials that are equivalent to the axioms considered by Ciabattoni. This allows the use of the rich meta-theory of linear logic in order to do automated proof search for a broad number os systems, including substructural, modal and paraconsistent logics.

3.9 Np Full System and Mimp-fol Association

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Abstract. Deal with larger proofs is a comum issue for automatic theorem provers. In order to reduce the time expended for presenting proofs, it can make use of a variety of logical systems, as well employ different ways to present the proof. Mimp-graph is a new structure to represent proofs through references rather than copy. Basically, mimp-graphs are special directed graphs whose nodes and edges are assigned with labels. Moreover we distinguish two parts, one representing the inferences of a proof, and the other the formulas. For the formula-part of a mimp-graph, we use formula graphs as a basis and consist only of formula nodes. For the inference-part of a mimp-graph we have the rule nodes (R- nodes) that are labelled with the names of the inference rules. The logic connectives and inference names may be indexed, in order to achieve a 1-1 correspondence between formulas (inferences) and their representations (names).The representation of a proof in mimp-graph requires fewer nodes than the tree or list representation of proofs. The capacity to represent any Natural Deduction proof is preserved. Another important advantage of a compact representation of graphs is that it allows to deduce some structural properties of proof-graphs, for example, based on a mimp-graph, it is easy to see an upper bound in the length of the reduction sequence to obtain a normal proof. It is the number of maximal formulas. Mimp-graph preserves the ability to represent any Natural Deduction proof and its minimal formula representation is a key feature of the mimp-graph structure, it is easy to distinguish maximal formulas and an upper bound in the length of the reduction sequence to obtain a normal proof. Sharing for inference rules is performed during the process of construction of the graph. This structure was initially developed for minimal propositional logic but the results have been extended to first-order logic preserving the major properties. The natural deduction system constituted only by the rules for implication introduction and implication elimination is

not complete with respect to the implicational fragment of classical propositional logic. A natural way to complete the system is through the addition of a new natural deduction rule corresponding to Peirce's formula. This system, named Np system, due the addition of the Peirce rule, creates a novel form of detour that destroys the sub-formula principle.This new form of detour is similar to a traditional detour for systems containing the classical absurd rule. Every derivation in the implicational fragment added with Peirce's rule can be transformed into a new derivation such that no application of Peirce's rule occurs above applications of implication introduction and implication elimination, this can be achieved by using the Seldin's normalization strategy. As an interesting corollary of Seldin's normalization strategy, the Np system has a form of Glivenko's theorem for classical implicational logic. Besides the advantage to be normalizable in a way that the proofs obtained can be splitted into intuitionistic part and a classical one, the Np systems shows to be effectively implemented in automatic theorem provers. Although, Np systems have not strong normalization. The association between Mimp-graph and Np system can be easily employed in automatic theorem provers. In this paper, we extend a association already done between mimp-graph and Np system for a more complex one evolving first order logic via Np full system and mimp for first order logic (mimp-fol). In particular, we present a strong normalization process. That is, we show that adding mimp-fol in Np full system the strong normalization is obtained instead of a weak and all that is achieved without loss of any Np system property, in particular the Glivenko's theorem still remains valid. [2]
Keyword: Np Full System. Mimp-fol. Normalization

[2] The authors gratefully acknowledge the partial financial support by CAPES, CNPq, UFG/RC and FAPERJ.

3.10 Subexponential and concurrent constraint programming

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Abstract. In previous works we have shown that linear logic with subexponentials (SELL), a refinement of linear logic, can be used to specify emergent features of concurrent constraint programming (CCP) languages, such as preferences and spatial, epistemic and temporal modalities. In order to do so, we introduced a number of extensions to SELL, such as subexponential quantifiers for the specification of modalities, and more elaborated subexponential structures for the specification of preferences. These results provided clear proof theoretic foundations to existing systems.

In this talk we answer positively the following question: can the proof theory of linear logic with subexponentials contribute to the development of new CCP languages? For that, we propose a CCP language with the following powerful features: 1) computational spaces where agents can tell and ask preferences (soft-constraints); 2) systems where spatial and temporal modalities can be combined; 3) shared spaces for communication that can be dynamically established; and 4) systems that can dynamically create nested spaces. In order to provide the proof theoretic foundations for such a language, we propose a unified logical framework (SELLSU) combining the extensions of linear logic with subexponentials mentioned above, and showing that this new framework has interesting proof theoretical properties such as cut-elimination and a sound and complete focused proof system.

3.11 Semi-BCI-Algebras

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Abstract. BCI-algebras are related to formal systems of Fuzzy Logic. They model the so called BCI-logics which are based on Curry combinators (B) $xyz.x(yz)$, (C)$xyz.xzy$ and (I) $x.x$. They are algebras of the form $<A, \ominus, \bot>$ which satisfy the following properties:

BCI-1 $((x \ominus y) \ominus (x \ominus z)) \ominus (z \ominus y) = \bot$

BCI-2 $(x \ominus (x \ominus y)) \ominus y = \bot$

BCI-3 $x \ominus x = \bot$

BCI-4 $x \ominus y = \bot and y \ominus x = \bot \longleftrightarrow x = y$

A BCI-algebra is called BCK-algebra, whenever it also satisfies:

BCK-1 $\bot \ominus x = \bot$

On any BCI-algebra it is possible to define a partial order "≤" as:

$$x \leq y iff x \ominus y = \bot$$

In this talk we will propose a generalization for such algebras, in order to capture some phenomena of Lukasiewcz Interval implications. Moreover, we show how Interval versions of MV-algebras can be obtained from such structure.

3.12 Minimal Paradefinite Logics for Reasoning with Incompleteness and Inconsistency

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Abstract. Paradefinite ('beyond the definite') logics are logics that can be used for handling contradictory or partial information, and so they have the following two properties:

- *Paraconsistency* [da Costa 1974]: The ability to properly handle contradictory data by rejecting the principle of explosion, according to which any proposition can be inferred from an inconsistent set of assumptions.
- *Paracompleteness*: The ability to properly handle incomplete data by rejecting the law of excluded middle, according to which for any proposition, either that proposition is 'true' (i.e., known) or its negation is 'true'.

Apart of these two primary requirements, a 'decent' logic for reasoning with indefinite data should have some further characteristics, like being expressive enough, faithful to classical logic as much as possible (in the sense that entailments in the logic should also hold in classical logic), and having some maximality properties (which may be intuitively interpreted by the aspiration to retain as much of classical logic as possible, while still preserving paraconsistency and paracomleteness).

In this work we are interested in the 'simplest' paradefinite logics (in terms of the number of the truth values of their semantics) that have the above properties. Obviously, two-valued logics are not adequate for this, as they cannot handle either of the two types of uncertainty. Likewise, three-valued logics can be used for handling just one type of uncertainty (see, e.g., [Avron 1991]), but they cannot simultaneously handle both of them. On the other hand, as shown e.g. in [Belnap 1977] and [Arieli and Avron 1998], four truth values *are* enough for reasoning with incompleteness and inconsistency.

Our study largely extends the work on 4-valued logics mentioned above. It provides a characterization of 4-valued paradefinite logics, examines these logics according to the criteria in Arieli and Avron and Zamansky [2011], investigates their relative strengths, and introduces corresponding proof systems.

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3.13 Toward of Nondeterministic Interval-Valued Fuzzy Logics: the case of the Negations

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(joint work with Regivan Santiago and João Marcos)

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Abstract. Fuzzy set theory was introduced as a mathematical framework to deal with approximate reasoning. The main idea is to consider truth degrees (values in the unit interval) instead of the classical Boolean values in order to capture vague concepts and reasoning. Since Zadeh's seminal work [Zadeh 1965] many extensions of fuzzy set theory have been proposed [Bustince et al 2015]. In particular, Torra in [Torra 2010] and [Torra and Narukawa 2009] proposes an extension of fuzzy sets called Hesitant Fuzzy Sets (HFS) where the membership is a set of values in the interval [0,1]. In [Wei Zhao and Lin], HFS were extended by considering a set of intervals instead of a set of values. In [Bedregal et al 2014a], the notion of Typical Hesitant Fuzzy Sets (THFS) is introduced for the extension where the membership is a finite and nonempty set of values in [0,1], which is in fact assumed in most constructions and applications of HFS. Analogously, in this work we introduce the notion of Typical Interval-Valued Hesitant Fuzzy Sets (TIVHFS).

In fuzzy logics, Zadeh suggests in [Zadeh 1965] a way of obtaining the complement of a fuzzy set by means of the notion of negation for fuzzy degrees of truth. In the following years, several other fuzzy negations were proposed. It was Trillas in [Trillas 1979] who introduced a general notion of fuzzy negation as a decreasing function $N : [0,1] \to [0,1]$ such that $N(0) = 1$ and $N(1) = 0$. Nowadays, Trillas's notion of fuzzy negation is the one used by the fuzzy logic community.

Fuzzy negations are an important mathematical tool in approximate reasoning as well as in decision making. This fundamental role of fuzzy negation has led to the introduction of an analogous notion for several extensions of fuzzy logics, as for example for typical hesitant fuzzy elements (HFE) [Bedregal et al 2014a, Bedregal et al 2014b, Santos et al 2014]. Let $\mathbb{U} = \{[x,y] : 0 \leq x \leq y \leq 1\}$ and $\mathcal{P}(\mathbb{U})$ the powerset of $\mathbb{U}$. Here, we consider the notion of fuzzy negation on $\mathbb{IH} = \{X \in \mathcal{P}(\mathbb{U}) : X \text{ is finite and non-empty}\}$, called Typical Interval-Valued Hesitant Fuzzy Negations (TIVHFN). For that, we also

introduce a partial order on $\mathbb{IH}$ that extends the partial order $\leq_{xx}$ introduced in [Xu and Xia 2011] and formalized in [Santos et al 2014].

On ther hand, according to Arnon Avron and Anna Zamansky [Avron and Zamansky 2011]:

> "The principle of truth-functionality (or compositionality) is a basic principle in many-valued logic in general, and in classical logic in particular. According to this principle, the truth-value of a complex formula is uniquely determined by the truth-values of its subformulas. However, real-world information is inescapably incomplete, uncertain, vague, imprecise or inconsistent, and these phenomena are in an obvious conflict with the principle of truth-functionality. One possible solution to this problem is to relax this principle by borrowing from automata and computability theory the idea of non-deterministic computations and apply it in evaluations of truth-values of formulas. This leads to the introduction of non-deterministic matrices (Nmatrices): A natural generalization of ordinary multi-valued matrices, in which the truth-value of a complex formula can be chosen non-deterministically out of some non-empty set of options."

Definition 1 *A non-deterministic matrix, Nmatrix or NM for short, for a propositional language $\mathcal{L}$ is a tuple $\mathcal{M} = \langle \mathcal{V}, \mathcal{D}, \mathcal{O} \rangle$ such that:*

1. *$\mathcal{V}$ is a non-empty set of truth values;*
2. *$\mathcal{D}$ is non-empty proper subset of $\mathcal{V}$; and*
3. *$\mathcal{O}$ is the set of functions $\tilde{\diamond}$ from $\mathcal{V}^n$ to $\mathcal{P}(\mathcal{V}) - \{\emptyset\}$, where $\diamond$ is a n-ary connective in $\mathcal{L}$.*

An NM in which $\mathcal{V} = \mathbb{U}$ and $\mathcal{D}$ is such tha if $X \in \mathcal{D}$ and $X \leq Y$ then $Y \in \mathcal{D}$, where $\leq$ is the usual order on intervals, will be called **interval-valued fuzzy NM** (IVFNM) and a NM such that the values of the connectives in $\mathcal{O}$ are finite sets will be called **finitely non-deterministic matrix**, **FNM** for short.

In this contribution, we introduce a **non-deterministic interval valued fuzzy negation for interval-valued fuzzy FNM**, as a function of the form $N : [0, 1] \rightarrow \mathbb{IH}$ which satisfies $N(0) = \{[1, 1]\}$, $N(1) = \{[0, 0]\}$ and $N(y) \leq_{xx} N(x)$ whenever $x \leq y$. Each TIVHFN based on Xu-Xia order $\mathcal{N}$ determines the non-deterministic negation $N(x) = \mathcal{N}(\{x\})$. We study several classes of nondeterministic negations and explore how to relate these to fuzzy negations.

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3.14 Bivaluations and generalized analyticity

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Abstract. According to Suszko's reduction theorem every Tarskian logic may be characterized by a two-valued semantics. Despite the result, Suszko never presented a constructive procedure to transform a many-valued semantics into a two-valued one. In [Caleiro Carnielli], an effective algorithmic procedure to carry out this task is presented for any finite-valued semantics. Many advantages are guaranteed by such procedure, for instance, its constructive character allows one to obtain a bivalent description of each operator from the original finite-valued semantics. Moreover, the information obtained from the bivalent description define two-signed tableaux systems with nice computational properties such as analyticity and also provide a decision procedure. The two-valued semantics with the properties presented in the format described in the paper is called **gentzenian semantics**. In [Avron 2009], Arnon Avron confront semantics given in terms of bivaluations with the usual many-valued matrix semantics approach. According to him, the crucial advantage of many-valued matrices is the *analyticity* property. Analyticity guarantees that every partial valuation over a set of closed formulas can be extended to a total valuation. Thus, if one wants to decide if some formula ϕ follows from a given set of formulas Γ, it is enough to assign values only to the relevant formulas, their immediate subformulas. It is well known however that the bivalent semantics obtained via Suszko's reduction procedure lacks such property. Therefore, based on a generalized notion of analyticity presented in [Caleiro Marcos and Volpe] and the technical machinery exhibited in [Caleiro Carnielli], the purpose of our work is to present a wider notion of gentzenian semantics, the so-called **quasi-gentzenian semantics**. We will show that in a quasi-gentzenian semantics it is possible to define a broader notion of analyticity. Moreover, some logics non-charaterizable by finite-valued matrices may be characterized in a quasi-gentzenian semantics.

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3.15 Semantic proofs of cut-elimination

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Abstract. There are two main approaches to proving cut-elimination: the syntactic and the semantic. A syntactic proof usually has the advantage of providing an algorithm for transforming a given proof to a cut-free one, while semantic proofs only demonstrate the admissibility of the cut-rule. On the other hand syntactic proofs of cut-elimination tend to be rather complicated, very difficult to follow and check, and very seldom given in full. Therefore in most cases semantic proofs are much more reliable. Another advantage of the semantic approach is that frequently it allows to prove cut-admissibility simultaneously to a whole family of logics. In this talk we present several examples of these claims, Including:
1) GL (Goedel-Loeb provability logic)
2) The hypersequential calculus for the semi-relevant logic RM
3) Families of logics based on finite-valued (non-deterministic) semantics.

3.16 Combined logics: characterizing mixed reasoning and applications

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Abstract. Fibring is a powerful mechanism for combining logics, and an essential tool for designing and understanding complex logical systems. Given two logic systems $\mathcal{L}_1$ and $\mathcal{L}_2$, their fibring $\mathcal{L}_1 \bullet \mathcal{L}_2$ is the smallest logical system for the combined language which contains both $\mathcal{L}_1$ and $\mathcal{L}_2$. That is, given consequence relations for $\mathcal{L}_1$ and $\mathcal{L}_2$, their fibring is the smallest consequence relation over the combined language that extends the two. From the point of view of the Hilbert systems, this corresponds precisely to putting together the Hilbert systems of the two logics, allowing instantiations with formulas in the joint signature along the derivations. In this presentation we give a full characterization of the consequence relation that emerges from fibring in the particular case where the logics being combined do not share any connectives. We show the power of this tool by presenting various applications on important problems regarding combined logics: (1) conservativity, (2) decidability and complexity, and (3) the preservation of many-valuedness.

Regarding (1), we provide a full characterization of the conditions under which the fibring of two logics without shared connectives is a conservative extension of both logics. Although this result does not apply to fibring with shared connectives, we have also developed a method based on translations whose scope goes well beyond.

Concerning (2), we take advantage of the description of the mixed patterns of reasoning to extract a decision procedure for the fibred logic that uses only the decision procedures for the component logics, and analyze its complexity. A full characterization of the decidability of fibred logics without shared connectives follows. In particular, the existence of this algorithm implies that fibring of logics with disjoint signatures preserves decidability. Again, although the results are limited to logics obtained by fibring syntactically disjoint fragments, we discuss how having this result may still be helpful regarding logics that can be presented as the fibring of disjoint fragments plus (a finite number of) interaction axioms.

Finally, with respect to (3), we show that our result can be useful in understanding the semantics of fibring. Using our characterization, we can show that fibring (even without shared connectives) does not preserve finite-valuedness nor many-valuedness.[3]

[3] The authors thank the support of EU FP7 Marie Curie PIRSES-GA-2012-318986 project GeTFun: Generalizing Truth-Functionality, and of FEDER/FCT project PEst-OE/EEI/LA0008/2013. Sérgio Marcelino also acknowledges the FCT postdoc grant SFRH/BPD/76513/2011

3.17 A hybrid logic that allows [local] inconsistencies

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Abstract. The aim of this talk is to present a paraconsistent version of hybrid logic. The world's semantics with nominals for this logic allows (local) inconsistencies without trivialization, and this is obtained through a model with two distinct valuations: one for positive literals, and another for negative literals. The existence of Robinson diagrams enables the representation of models as sets of hybrid formulas and thus it makes it possible to evaluate a model with regard to its number of inconsistencies. We will also discuss proof-theoretical aspects of this logic.

3.18 Should a new hierarchy in AAL be build?

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Abstract. In [Font 2009], Font discusses two notions of many-valued consequence relations, the truth-preserving one and the one that preserves degrees of truth. The former divides in two sets the universes of its algebraic models differentiating the designated and the undesignated elements, therefore giving a bivalent treatment to the truth-values, while the latter takes into account all the truth values of its models and how they relate to each other. In that paper, a general framework is investigated for the algebraization of finitary logics that preserve degrees of truth. Although the author requires only that the universe be accompanied with an ordering, we only find in the literature studies restricted to lattice-like structure, in general, and to meet-semilattices with a maximum element or to residuated lattices, in particular. Such restriction is justified in [Font 2015], a paper that surveys the relation between both notions of many-valued consequence relations.

The AAL community has created the Leibniz and the Frege hierarchies, each with a different character. As surveyed in [Font 2015], the classification in these hierarchies of both notions of many-valued consequence relations has already been done. Noting that the logics that preserve degrees of truth have the particular characteristic of having ordered-algebraic models, we raise the question that maybe a new hierarchy with respect to the type of order must be built. Trying to answer this question we propose the hierarchy showed in Figure 3.1.

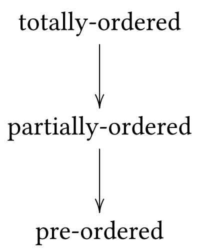

Figure 3.1: The classes in the order hierarchy. The arrows mean “included in” or “implies”.

We believe that none of the proposed levels are empty. For instance, members of the ALOPHIS group and “friends” studied in [Bou et al. 2010, Paoli et al. 2011] two logics associated to a pre-order (obtained as a generalization of the work done in [Berman and Blok 2006]). Perhaps the logics with a meet-semilattice model belong to the partially-ordered class and the logics associated with linearly-ordered MTL algebras are classified as part of the strongest level.

Since one of the main purposes of AAL is to study the relations in between the metalogical properties of logics and purely algebraic properties of their classes of algebraic models, this hierarchy can only become interesting if it serves this purpose. We believe that this can open a fruitful line of investigation, since there has already been studies made about the algebraic properties of ordered structures in the context of universal algebra (as in [Berman and Blok 2006]) and the metalogical properties or abstract logics are well-known, so what remains to be better understood is the relation between them.

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3.19 Many-valued modal logic over residuated lattices via duality

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Abstract. One of the latest and most challenging trends of research in non-classical logic is the attempt of enriching many-valued systems with modal operators. This allows us to formalize reasoning about vague or graded properties in those contexts (e.g., epistemic, normative, computational) that require the additional expressive power of modalities. This enterprise is thus potentially relevant not only to mathematical logic, but also to philosophical logic and computer science.

A very general method for introducing the (least) many-valued modal logic over a given finite residuated lattice is described in Bou [et al. 2011]. The logic is defined semantically by means of Kripke models that are many-valued in two different ways: the valuations as well as the accessibility relation (viewed as the associated characteristic function) among possible worlds are both many-valued. The work in Bou [et al. 2011] also shows that providing complete axiomatizations for such logics, even if we enrich the propositional language with a truth-constant for every element of the given lattice, is a non-trivial problem, which has been only partially solved to date.

In this presentation we report on ongoing research in this direction, focusing on the contribution that the theory of natural dualites Clark and Davey [1998] can give to this enterprise. We show in particular that duality allows us to adapt the method used in Bou [et al. 2011] to prove completeness with respect to local modal consequence, obtaining completeness for global consequence, too (a problem that, in full generality, was left open in in Bou [et al. 2011]). Besides this, our study is also a contribution towards a better general understanding of quasivarieties of (modal) residuated lattices from a topological perspective.

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3.20 Axiomatizing normal modal (paraconsistent) logics

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Abstract. In [Béziau 2002] Béziau showed how to define a paraconsistent logic, in the signature $\{\wedge, \vee, \supset, \smile\}$, where $\smile$ is a paraconsistent negation, from a modal logic in the signature $\{\wedge, \vee, \supset, \sim, \Diamond, \Box\}$. But it is not clear for him what is the adequate axiomatization of the defined paraconsistent logics. In [Béziau 2006] he solved this problem, but only for the logic S5. In [Marcos 2005] Marcos proposed an alternative axiomatization to the normal modal logic K also based on the signature $\{\wedge, \vee, \supset, \smile\}$. The axiomatization consists in adding to any complete set of rules and axioms for positive classical logic the following rules and axioms:

(A1) $\vdash \smile(\varphi \wedge \psi) \supset (\smile\varphi \vee \smile\psi)$
(R1) If $\vdash \varphi \supset \psi$, then $\vdash \smile\psi \supset \smile\varphi$
(R2) If $\vdash \varphi$, then $\vdash \smile\varphi \supset \psi$

The author claimed that the former axiomatization is complete with respect the usual Kripke semantics and still suggest axiomatizations based in this minimal axiomatization to cover other logics. But the results was not proven. In [Dodó 2013] we showed a completeness results for these logics. The proofs made use of a classical negation $\sim$, that can be defined in this logic in the following way:

$$\sim\gamma \stackrel{def}{=} \gamma \supset \smile(\alpha \supset \alpha)$$

.

In this work we shall present a completeness proof too. But this time the proof does not make reference to a classical negation.

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3.21 How Kelsenian Jurisprudence and Intuitionistic Description Logic help to avoid Contrary-to-Duty paradoxes in Legal Ontologies

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Abstract. Classical Logic has been widely used as a basis for ontology creation and reasoning in many knowledge specific domains. These specific domains naturally include Legal Knowledge and Jurisprudence. As in any other domain, consistency is an important issue for legal ontologies. However, due to their inherently normative feature, coherence (consistency) in legal ontologies is more subtle than in other domains. Consistency, or absence of logical contradictions, seems more difficult to maintain when more than one law system can judge a case. This is called a conflict of laws. There are some legal mechanisms to solve these conflicts, some of them stating privileged fori, other ruling jurisdiction, etc. In most of the cases, the conflict is solved by admitting a law hierarchy or a law precedence.

Even using these mechanisms, coherence is still a major issue in legal systems. Each layer in this legal hierarchy has to be consistent. Since consistency is a direct consequence of how one deals with logical negation, negation is also a main concern of legal systems. Deontic Logic, here considered as an extension of Classical Logic, has been widely used to formalize the normative aspects of the legal knowledge. There is some disagreement on using deontic logic, and any of its variants, to this task. Since a seminal paper by Alchourron [Alchourron and Martino 1990], the propositional aspect has being under discussion. In this case, *laws* are not to be considered as propositions. This is in fully agreement with Hans Kelsen jurisprudence [Kelsen 1991]. On a Kelsenian approach to Legal Ontologies, the term "Ontologies on laws" is more appropriate than "Law ontologie". In previous works we showed that Classical logic is not adequate to cope with a Kelsenian based Legal Ontology. Because of the ubiquitous use of Description Logic for expressing ontologies nowadays, we developed an Intuitionistic version of Descripion Logic particularly devised to express Leagal Ontologies. This logic is called iALC [Haeusler et al. 2010a, de Paiva et al. 2010, Haeusler et al. 2010b].

Our system is based on the framework for intuitionistic modal logics proposed by Simpson [Simpson 1993] and called IML (intuitionistic modal logic).

These modal logics arise from interpreting the usual possible worlds definitions in an intuitionistic meta-theory. We also use ideas from [Braüner and de Paiva 2006], where the framework IHL, for *intuitionistic hybrid logics* is introduced. Hybrid logics add to usual modal logics a new kind of propositional symbol, the nominals, and also the so-called satisfaction operators. A nominal is assumed to be true at exactly one world, so a nominal can be considered the name of a world. If x is a nominal and X is an arbitrary formula, then a new formula $x\colon X$ called a satisfaction statement can be formed. The part $x\colon$ of $x\colon X$ is called a satisfaction operator. The satisfaction statement $x\colon X$ expresses the fact that the formula X is true at one particular world, namely the world at which the nominal x is true. Out of these tightly connected systems of intuitionistic modal IML and hybrid logics IHL, we want to carve out our logic. iALC concepts are described as:

$$\begin{aligned} C, D ::= \quad & A \mid \bot \mid \top \mid \neg C \mid C \sqcap D \mid C \sqcup D \mid C \sqsubseteq D \\ & \mid \exists R.C \mid \forall R.C \\ F ::= \quad & C \sqsubseteq D \mid a\colon C \mid aRb \end{aligned}$$

where A stands for an atomic concept, R for an atomic role, F for a formula and a, b for individuals (in hybrid logic reading, nominals). We could have used distinct symbols for subsumption of concepts and the subsumption concept constructor but this would blow-up the calculus presentation. This syntax is more general than standard ALC since it includes subsumption $\sqsubseteq$ as a concept-forming operator.

A constructive interpretation of iALC is a structure $\mathcal{I}$ consisting of a non-empty set $\Delta^{\mathcal{I}}$ of entities in which each entity represents a partially defined individual; a refinement pre-ordering $\preceq^{\mathcal{I}}$ on $\Delta^{\mathcal{I}}$, i.e., a reflexive and transitive relation; and an interpretation function $\cdot^{\mathcal{I}}$ mapping each role name R to a binary relation $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ and each atomic concept A to a set $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ which is closed under refinement, i.e., $x \in A^{\mathcal{I}}$ and $x \preceq^{\mathcal{I}} y$ implies $y \in A^{\mathcal{I}}$. The interpretation $\mathcal{I}$ is lifted from atomic $\bot, A$ to arbitrary concepts via:

$$
\begin{aligned}
\top^{\mathcal{I}} &=_{df} \Delta^{\mathcal{I}} \\
(\neg C)^{\mathcal{I}} &=_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}.x \preceq y \Rightarrow y \notin C^{\mathcal{I}}\} \\
(C \sqcap D)^{\mathcal{I}} &=_{df} C^{\mathcal{I}} \cap D^{\mathcal{I}} \\
(C \sqcup D)^{\mathcal{I}} &=_{df} C^{\mathcal{I}} \cup D^{\mathcal{I}} \\
(C \sqsubseteq D)^{\mathcal{I}} &=_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}.(x \preceq y \text{ and } y \in C^{\mathcal{I}}) \Rightarrow y \in D^{\mathcal{I}}\} \\
(\exists R.C)^{\mathcal{I}} &=_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}.x \preceq y \\
&\quad \Rightarrow \exists z \in \Delta^{\mathcal{I}}.(y,z) \in R^{\mathcal{I}} \text{ and } z \in C^{\mathcal{I}}\} \\
(\forall R.C)^{\mathcal{I}} &=_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}.x \preceq y \\
&\quad \Rightarrow \forall z \in \Delta^{\mathcal{I}}.(y,z) \in R^{\mathcal{I}} \Rightarrow z \in C^{\mathcal{I}}\}
\end{aligned}
$$

In this work we show how the iALC avoids some Contrary-to-duty paradoxes, as Chisholm's paradoxes and its variants. For each of these paradoxes we provide an iALC model. Finally we discuss the main role of the intuitionistic negation in this issue, finding out that its success may be a consequence of its paracomplete logical aspect. An investigation on the use of other paracomplete logics in accomplish a logical basis for Kelsenian legal ontologies is highly motivated.

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4 2nd FILOMENA: Philosophy, Logic and Analytical Metaphysics

Dates: Sep 2-4, 2015

The second edition of the **FILOMENA Workshop** (**FI**losofia, **LO**gica e **ME**tafísica a**NA**lítica), promoted by the **Group on Logic and Formal Philosophy** from the UFRN, has the purpose of gathering logicians working at the intersection of Logic and Metaphysics, through the application of formal methods in Philosophy. Logic, while initially considered as a branch of Philosophy, has outgrown its original purposes and found connections with other areas of Philosophy, such as Philosophy of Language, Philosophy of Mathematics, Philosophy of Science and Philosophy of Mind. Since its modern development, Logic has proved to be a powerful tool for analyzing different philosophical theories, as well as their foundations and implications; moreover, the birth and development of non-classical logics has expanded its domain of application much beyond the dreams of its progenitors.

Topics of interest for our Workshop include, but are not limited to:

- Modal metaphysics
- Reference and descriptions
- Philosophical topics in non-classical logics
- Truth-values
- Logical consequence
- Logical pluralism x logical monism
- Logic and metaphysical neutrality
- Paradoxes

The first edition of FILOMENA, held at UFRN in September 2014, counted with 15 contributed talks and a round-table on "The philosophy of contradictions", with the following participants: Hitoshi Omori (CUNY), Peter Verdée (KU-Leuven), Marcos Silva (UFC), Chair: João Marcos (UFRN).

4.1 Dialetheism and Dual-Valuation Logics

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Abstract. Dialetheism is the thesis that some contradictions are true. To accommodate their thesis dialetheists require a logic that is both paraconsistent and allows for contradictions, formally conceived as formulae of the form $A \wedge \neg A$ to be assigned the truth-value true. Logics that fulfil both of these conditions are *dialetheic* logics. In addition to requiring a dialetheic logic, for philosophical reasons, dialetheists prefer logics that both respect the normal semantics for conjunction and negation, that is $v(A \wedge B) = min\{v(A), v(B)\}$ and $v(\neg A) = 1 - v(A)$, and conceive of propositional valuations as relations between propositional parameters and the set of truth-values, rather than functions. Unfortunately for the dialetheist, every current dialetheic logic that fulfils these further two conditions can be found to suffer from three rather damaging criticisms: they are unable to express the meaningful concepts of 'true only' and 'false only', they are unable to accommodate quasi-valid inferences (inferences which are classically valid but dialetheic invalid), and they commit dialetheists to the impossibility of the actual world. In this talk I introduce a new family of logics, dual-valuation logics, and explain how one glutty member of the family, **gD-V**, can provide a solution to these three criticisms for the dialetheist.

Dual-valuation logics are logics that conceive of the total valuation of a propositional parameter in a zero-order logic as constituted by two relations, ε^+ and ε^-, rather than one. The first relation, ε^+, is a *valuation* relation and is the same as the *valuation* relations found in other logics with relational semantics. The second relation, ε^-, found in dual-valuation logics, however, rather than symbolising the logic's *valuation* relation, symbolises the logic's anti-valuation relation. The truth-values that a proposition p has the relation ε^+ to dictates p's *valuation* set, and the truth-values that p has the relation ε^- to dictates its *anti-valuation* relation. Just as certain objects are in the extension of a predicate P, and other objects are in P's anti-extension, so truth-values are either in the valuation set of a proposition p, or its anti-valuation set. As is normally the case, valuations are relations from propositional parameters to the set of truth-values true, false, and anti-valuations are similarly relations from propositional parameters to the set of truth-values.

Every member of the dual-valuation family places two restrictions upon the membership conditions of the valuation and anti-valuation sets for a proposition. For any proposition p and truth-value t: 1) Either $p\varepsilon$+t or $p\varepsilon$-t, and 2) It's not the case that both $p\varepsilon$+t and $p\varepsilon$-t. Thus, it's an assumption of all dual-valuation logics that the valuation and *anti-valuation* sets partition the set of truth-values for every propositional parameter. Consequently, these restrictions placed on the valuation and anti-valuation sets for a proposition ensure that the metatheory of all dual-valuation logics behaves consistently. Any other restrictions which are placed upon the membership conditions of the valuation and anti-valuation sets for a proposition are dependent upon the individual logics within the dual-valuation family.

In this talk we concentrate on one member of the dual-valuation family, the glutty but non-gappy **gD-V**, which has normal semantics for conjunction and negation. In **gD-V**both the truth-values true and false can be members of a propositional parameter's valuation set, however the valuation set for a propositional parameter must be non-empty. Thus, although a propositional parameter p can have the valuation relation to both true and false, p can only have the anti-valuation relation to either true or false. Consequently, there are three permissible total valuations for a propositional parameter in **gD-V**: 1) $p\varepsilon$+1 and $p\varepsilon$-0; 2) $p\varepsilon$+0 and $p\varepsilon$-1; and 3) $p\varepsilon$+1, $p\varepsilon$+0, and $p\varepsilon\varnothing$.

We demonstrate that **gD-V** is a *Logic of Formal Inconsistency* (**LFI**), by being able to recapture classical validity through consistency assumptions in the logic's object language, and that unlike other well-known **LFI**'s **gD-V** has a 'consistent truth' and 'consistent falsity' operator. We then move on to show that, given these facts, **gD-V** can both express the concepts of 'true only' and 'false only' within its semantics, as well as accommodating quasi-valid inferences. We then show that a modal extension of **gD-V**, **gD-Vm**, blocks the dialetheist's commitment to the impossibility of the actual world while possessing other interesting formal properties, such as failing to include formulae of the form $\neg\Diamond(A \wedge \neg A)$ as theorems, even though the logic both respects the normal semantics for conjunction and negation, and adheres to an intuitive modal semantics. It's argued that this latter result suggests that gD-V provides the dialetheist with the resources necessary to express, within her logic's object language, disagreement with the classical logician over the truth of contradictions, something that current dialetheic logics fail to provide.

We end by showing that, irrespective of **gD-V**'s expressive power, it is inadequate for the dialetheist who wishes to provide a comprehensive dialetheic solution to the self-referential paradoxes, but that this fact shouldn't detract from **gD-V**'s utility and novelty.

4.2 Logics of Formal Inconsistency with a transparent truth predicate

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Abstract. The logics of formal inconsistency (LFIs) are powerful paraconsistent logics that encode classical logic and permit to fix an interesting distinction between contradictions and inconsistencies. These systems, introduced among others in [Carnielli and Marcos 2002] and later developed in [Carnielli et al. 2004], [Marcos 2005] and [Carnielli et al. 2007], internalize the metatheoretical notion of consistency, expressing it in the object language. Hence one can isolate contradictions in such a way that the application of the principle of explosion is restricted to consistent sentences only, thus avoiding triviality. In this talk, we are going to explore how to talk about truth in LFIs. Specially, focusing on the paradoxes of the Strengthened Liar and Curry's, formulated in a language that contains transparent truth and the consistency operator. In the literature about formal theories of truth, the idea of expressing all the theoretical and metatheoretical notions of some theory in the object language of that theory itself is commonly referred to as the attempt to have "semantically closed languages". Here, we will show that in a family of non-infectious paraconsistent logics one is not capable of avoiding trivialization in the presence of these sentences. Some logics of this sort are Priest's Logic **LP** augmented with a consistency operator and a truth predicate. The resulting theory of truth **LPc**+ is trivial. We know this because its paracomplete dual, Strong Kleene Logic $\mathbf{K_3}$ with its language augmented in the same way is known to be trivial in the presence of a truth predicate. We also show that some recently developed LFIs are trivial in the presence of a truth predicate. In this way, one cannot express transparent truth in a language that is capable to talk about consistency. Therefore, these logics cannot be semantically closed.

In the second part of the talk, we will introduce a family of infectious paraconsistent logics, i.e. paraconsistent logics where all operations that take some non-classical value (paradigmatically, "both true and false") as an input give a non-classical value as its output. We will show that logics belonging to this collection are capable of avoiding triviality even in presence of

transparent truth and consistency. We argue that this fact obtains mainly because in these systems a particular sort of strong negation –involved in the formulation of the Strengthened Liar paradox– is not definable. Thus, this part of the paper delves into the following question: which are the minimal and maximal LFIs of that family? We first discuss the case of **PWKc**+, the paraconsistent version of Weak Kleene Logic augmented with a consistency operator. This logic is proven to be an interesting logic of both formal inconsistency and truth, since it is not subject to trivialization neither via The Liar Paradox (as was independently hinted by H. Omori and R. Cuini in [Omori and Ciuni 2014]), nor via Curry's Paradox, as we show in this paper. Finally, we point out the existence of some weaker LFIs that might handle well the notions of consistency and truth, which indicates that **PWKc**+ is not the smallest of the set of paraconsistent logics in question. Furthermore, the existence of these logics of formal inconsistency and truth is a clear sign that the project of having semantically closed languages is very much alive, precisely thanks to the formal tools provided by the work in LFIs.

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4.3 Non-Contractive Paralogics

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Abstract. Since Alfred Tarski's work on truth in the 30s and 40s, it is well-known that it is not possible to define truth within a classical theory powerful enough to express its own syntax, such as Peano arithmetic. Tarski imposed an adequacy condition on any definition of truth, namely, that all the instances of the *T-schema* must follow from it. This means that for every sentence ϕ of the language of the theory it must hold that

$$(\textit{T-schema}) \qquad Tr\langle\phi\rangle \text{ if and only if } \phi$$

where $\langle\phi\rangle$ is a name for the sentence ϕ and Tr is the truth predicate. Tarski's undefinability theorem shows that adding the *T-schema* to any classical theory capable of expressing its own syntax results in an inconsistency, in fact, in triviality. For we can construct a Liar sentence λ such that:

$$\lambda \text{ if and only if it is not the case that } Tr\langle\lambda\rangle.$$

Intuitively, λ says of itself that it is not true, which directly contradicts the *T-schema*.

Since then, different approaches have been proposed to avoid this difficulty. Roughly, either classical logic is rejected in favor of a weaker logic or the principles governing the truth predicate are restricted in some way. This paper is concerned with the former project.

There are basically two ways of weakening classical logic for this purpose. We can either modify the meaning of one or more of the usual logical constants, or we can directly modify the meaning of the logical consequence relation. Typically, the first sort of approach is called 'operational' and the second 'substructural'.

There have been several proposals in both directions. The operational approach has received a lot of attention in the last few decades. Among its recent advocates we find paraconsistent theorists, such as [Priest 2006] and [Beall 2009] and paracomplete theorists, such as [Field 2008], just to name a few. The substructural approach is much younger. Among its advocates we find non-transitive theorists, such as [Ripley 2012] and non-contractive theorists, such as [Zardini 2011] and [Mares and Paoli 2014], among others.

In this paper I focus on non-contractive theories of truth. Non-contractive theories reject one (or both) of the following structural rules (I assume familiarity with sequent calculi; Γ and Δ are multisets and $\Rightarrow$ stands for some form of entailment):

$$\text{L}C\ \frac{\Gamma, \phi, \phi \Rightarrow \Delta}{\Gamma, \phi \Rightarrow \Delta} \qquad\qquad \text{R}C\ \frac{\Gamma \Rightarrow \phi, \phi, \Delta}{\Gamma \Rightarrow \phi, \Delta}$$

My main goal is to lend some support to the idea that the non-contractive route is available to the supporters of paracomplete and/or paraconsistent logics as well. In other words, we shouldn't see the non-contractive approach as a rival to the operational approaches, or at least, there is no need to do so. The reason is that the failure of the structural Contraction rules is very much entangled with both the failure of Excluded Middle and the failure of Explosion. In general, theories that reject RC cannot prove $\Rightarrow \phi \vee \neg\phi$. And similarly, in general, theories rejecting LC cannot prove the sequent $\phi \wedge \neg\phi \Rightarrow$.

In this paper I argue in favor of this idea by defending a specific non-contractive theory of truth that is also paracomplete (although it is very easy to obtain a paraconsistent dual). From a conceptual point of view, the theory has, in my opinion, some advantages over other non-contractive theories that have been proposed in the literature. For one thing, since the theory is paracomplete, it gives a nice diagnosis of the Liar and other paradoxical sentences. In particular, both the Liar and its negation are rejected. Also, the theory only contains an additive conjunction and an additive disjunction, so there is no need to posit some form of ambiguity affecting the natural language expressions 'and' and 'or'. From a more technical point of view, I show that the theory can contain a naive truth predicate (i.e. a predicate satisfying the full *T-schema*), and that it can even contain a naive validity predicate (something that is not possible in purely operational approaches). I also offer a Cut-elimination proof which has as a corollary the theory's consistency.

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4.4 Understanding paraconsistent contradictions

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Abstract. Paraconsistent logics are usually characterized as systems that underlie inconsistent but non-trivial theories. In this regard, such systems are thought of as domesticating contradictions: one may have contradictions and still avoid triviality. That is revolutionary, in the sense that one may be seen as violating what was taken by Aristotelian tradition as the most fundamental law, the law of non-contradiction. Now, one of the challenges for a paraconsistentist is to explain the precise meaning of a paraconsistent negation, and how one may deal with "true" contradictions. On one extreme we have dialetheists, holding that there are true contradictions in reality (even if they are in the non-observable part of reality). On a rival deflationist approach, one may see contradictions as a feature of our theories. In this reading, contradictions are like a by-product of the incompleteness of our theories. They are not a feature of reality. Anyway, whichever path one chooses, a paraconsistent logic is said to be required. Despite the worry of paraconsistentists to make sense of contradiction, there is still another feature of paraconsistent logics that must be addressed: a paraconsistent negation is not what could traditionally be termed a "contradiction forming" operator. That is, a pair of propositions p and $\neg p$, where $\neg$ is a paraconsistent negation, is not a contradiction. In fact, paraconsistent negations are subcontrary-forming operators, in the sense that they generate a weaker kind of opposition, called subcontrariety (see Béziau [Béziau 2006]). Our efforts in this communication will be to put together such awkward pieces of information. On the one hand, if a paraconsistent negation is really only a subcontrary forming operator, why should we care about the meaning of contradictions? In particular, why should we dispute about whether contradictions are found in theories or in reality, given that contradictions are not even in issue? On the other hand, how to account for the application of paraconsistent logics in situations that reasonably seem to involve contradictions? Our thesis is that both situations may be perfectly accommodated. On the one hand, by analyzing examples of application of paraconsistent logics, one may reasonably claim that they really involve cases of subcontrariety. A typical example comes from group discussions. In a group, distinct members may have contradictory opinions, so that member 1 may claim p, while member 2 may claim $\neg p$. One may see

that as a contradiction, but taking into account the group, the sentences involved are "member 1 claims p" and "member 2 claims $\neg p$", which is really a subcontrariety. That is, by suitably interpreting the situation one introduces subcontrariety, allowing for the meaningful application of paraconsistent logics and evading direct contradictions. A similar situation holds for cases of inconsistent information. Generally, distinct sources S and R are responsible for the contradiction, so that we have "according to S, p is the case" and "according to R, $\neg p$ is the case". That is again a case of subcontrariety. What is to be welcomed by the paraconsistentist is that this move allows for the application of paraconsistent logics and avoids the need for understanding contradictions (see also Arenhart [Arenhart 2015]). Obviously, this is a radicalization of the so-called "epistemic interpretation" of contradictions, by Carnielli and Rodrigues [Carnielli and Rodrigues 2012]. The fact is that once one allows a context to come into the scene, there is no contradiction anymore. We shall investigate what kind of contexts C are suitable for the paraconsistentist, so that one may go from a straight contradiction as p and $\neg p$ to a subcontrariety of the form $C(p)$ and $C(\neg p)$. Obviously, not every context will work, and we shall present some cases. What is relevant is that this kind of move allows one to make sense of many applications of paraconsistent logics without requiring that we accept contradictions, and also, by taking seriously the fact that paraconsistent negations generate subcontrariety.

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4.5 Is complementary classical logic really a *logic*?

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(joint work with Walter A. Carnielli[1], and Achille C. Varzi[2])

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Abstract. Two deductive systems $\mathcal{S}$ and $\overline{\mathcal{S}}$, sharing a same language $\mathcal{L}$, are said to be *complementary* [Varzi 1990] just in case, for every formula α of $\mathcal{L}$:

$$\nvdash_{\mathcal{S}} \alpha \text{ if and only if } \vdash_{\overline{\mathcal{S}}} \alpha.$$

In other words, a system $\overline{\mathcal{S}}$ turns out to be complementary with respect to another system $\mathcal{S}$ if, and only if, it exactly proves the non-theorems of $\mathcal{S}$. The informal idea underlying the study of complementarity is that of characterizing a given system $\mathcal{S}$ by taking, so to speak, the negative of its picture. Together, the two systems provide a uniform complete characterization of a logic—a fact that can prove of theoretical interest especially in connection with decidability results (see e.g. [Skura 1991]).

Needless to say, the most interesting complementary systems are those involving well-know deductive systems, classical logic *in primis*. When $\mathcal{S}$ is propositional classical logic, its non-theorems admit of a straightforward semantic characterization (just consider those formulas for which there is at least one truth-value assignment that falsifies them). A proof-theoretical characterization, however, is more challenging, not least because $\overline{\mathcal{S}}$ must be paraconsistent (the non-theorems of $\mathcal{S}$ include all contradictions as well as mutually contradictory contingent formulas, such as p and $\neg p$). Łukasiewicz's refutation calculus can be seen as the first deductive system complementing classical logic [Lukasiewicz 1951]. More than twenty years later, Caicedo provided the first Hilbert-style calculus for complementary classical logic in [Caicedo 1978]. Other calculi of this sort were proposed by Varzi at the beginning of the 90s in [Varzi 1990][Varzi 1992]. Around the same years, Tiomkin [Tiomkin 1988] presented the first Gentzen-style sequent calculus for complementary classical logic with rules for negation and disjunction. This calculus was extended by Bonatti and Olivetti [Bonatti and Olivetti 2002] to include rules for the whole spectrum of classical connectives. Neither of these calculi, however, considers cut rules, thus excluding the possibility of

implementing a cut-elimination algorithm in a satisfactory way. Indeed, in complementary classical logic, the cut rule in its standard Gentzen formulation (cf. [Gentzen 1935]) is not admissible:

$$\frac{\Gamma, \alpha \vdash \Delta \qquad \Gamma' \vdash \Delta', \alpha}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\ cut.$$

The following example shows how a (classically) valid sequent can be obtained by cutting two (classically) non-valid sequents:

$$\frac{p \nvdash p \to q \qquad p \to q \nvdash p}{p \vdash p}\,.$$

In [Tiomkin 1988], Tiomkin reports a couple of rules, which he calls 'cuts for the unprovability', obtained by "reversing" the (additive) standard cut rule:

$$\frac{\Gamma \nvdash \Delta \qquad \Gamma \nvdash \Delta, \alpha}{\Gamma, \alpha \nvdash \Delta}\ cut \nvdash \qquad\qquad \frac{\Gamma \nvdash \Delta \qquad \Gamma, \alpha \nvdash \Delta}{\Gamma \nvdash \Delta, \alpha}\ \nvdash cut.$$

However, proof-theoretically such a denomination turns out to be improper, since both these rules preserve the subformula property.

In this contribution, we consider Bonatti and Olivetti's complementarization of the classical sequent calculus LK and show how to enrich it with some versions of the cut rule that are admissible in $\overline{\text{LK}}$. An efficient and simple normalization procedure is then provided, showing it to be strong normalizing (in the sense that any reduction strategy terminates) and strongly confluent. As is well-known, this latter fact implies uniqueness of the normal form [Girard and Lafont and Taylor 1989]. Moreover, unlike in LK, normalization in $\overline{\text{LK}}$ always induces a remarkable simplification of proofs. This confirms that the study of complementary systems is not purely a theoretical exercise and can have practical implications as well.

We conclude by addressing the following philosophical question: can $\overline{\text{LK}}$ be considered a *logic* to all intents and purposes? One could answer either way. On the one hand, $\overline{\text{LK}}$ is a deductive system that fully identifies, albeit negatively, the classical tautologies, hence it captures classical logic after all. On the other, its consequence relation is paraconsistent and clearly violates certain structural properties that may be regarded as constitutive of deductive rationality, such as reflexivity, monotonicity, and transitivity. Our inclinations lean towards an affirmative answer, and we stress the relevance of our proof-theoretical results to that effect.

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4.6 On logical pluralism and non-Tarskian entailment

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Abstract. In [Malinowski 1990], the Polish logician G. Malinowski proposed an alternative notion of logical consequence called $quasi$-consequence. Differently from the Tarskian notion of entailment, where valid inference means the preservation of truth from the premises to the conclusion, in q-entailment valid inference means that the conclusion is accepted whenever all premises are not rejected. Thus, it is not necessary that all premises should be accepted, we may have premises that are neither accepted nor rejected. Later on, the dual of q-entailment was proposed by Frankowski [Frankowski 2004] and baptized by him as $plausible$-entailment (shortly p-entailment), where valid inference means that the conclusion is not rejected whenever all premises are accepted.

The purpose of the present work is to explore the non-Tarskian notions of entailment presented above and confront their philosophical foundations with logical pluralism as presented by Beall & Restall in [Beall and Greg 2006]. The problem of what should count as an adequate ground for a notion of logical consequence, as well as what kinds of pluralism could be defended beyond the Tarskian paradigm of entailment, shall be adressed.

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4.7 How to be or not to be a logical pluralist

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Abstract. For the most of the history of philosophy, it was assumed that there was one correct logic, that of Aristotle, which developed further during the Middle Ages. In more recent history, the world has seen the development of other logics, such as intuitionistic, paraconsistent, relevant, and quantum logic. What should be done the face of this? Choose one logic and stick to it, or do you get to pick as many as you want? Logical pluralism is the thesis that there is more than one correct logic. But what does this mean exactly? In what context can more than one logic be accepted? The present talk plans on exploring different views of what it means to be a logical pluralist.

One such view is that of [Beall and Restall 2006], which defends that there can be more than one valid logical consequence relation within the same language. They define a logical consequence as having features: necessity (the truth of the premises necessitates the truth of the conclusion), normativity (it is not rational to reject a conclusion) and formality (here they accept different meanings for this: logic provides constitutive norms for thought, or is indifference of indistinguishable, or abstracts from semantic content). They explain validity in some logic in term of cases: "A valid argument is one whose conclusion is true in every case in which all its premises are true" (p. 23). This notion is not left very clear, and has received a lot of criticism.

Their pluralism sets out from the Generalized Tarski Thesis (GTT), which states that: "An argument is $valid_x$ if and only if, in every $case_x$ in which the premises are true, so is the conclusion" (p. 29). Logical pluralism is then the claim that there are at least two instances of GTT that gives a precisification of a logical consequence. In their book, they illustrate their pluralism with classical logic (in which the case is taken to be possible worlds), intuitionistic logic (in which the case is taken to be incomplete constructions) and relevant logic (in which the case is taken to be situations, that can be inconsistent). They use the terminology of "strong endorsment" and "weak endorsment" to explain how one might take two logics to be valid in the same context.

A critic of this view is Graham Priest, who in [Priest 2001] argues against Beal and Restall's pluralism in many fronts. In this same article, Priest distinguishes different kinds of pluralism, explaining why some of them are

interesting or not. When the topic is pure logic, that is, a well-defined mathematical structure, with proof-theory and so on, pluralism exists just because there are many different logics in this sense. Rivalry only arises when some logic is applied to some end, that is, when dealing with applied logic, in which a pure logic is interpreted into a theory of something. He calls "theoretical pluralism" the view that there maybe be different applied logics with constitute the theory of a same domain. This pluralism, however, is not "the hard question". The hard question is concerned with the application of logic in the analysis of reasoning. In [Priest 2006], on this point, he argues for monism.

Another kind of pluralism is that of Rudolf Carnap. His pluralism deals with the freedom to choose the formal language in which one wishes to evaluate arguments, such as first-order or second-order predicate logic. This kind of pluralism is dependent on the language. In [Cook 2010], Roy Cook argues that Beal and Restell's pluralism is dependant on the "logical consequence relation in natural language on intends to codify" (p. 500) in the same way that Carnap's pluralism depends on language, and thus it is not free of relativism.

These are just some views and criticisms among the many that can be found in the literature. This talk hopes to showcase different views on this matter so that one can choose how best to be or not to be a logical pluralist.

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4.8 From Commitment to Indifference: a critique of Quine's formal naturalism

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Abstract. In one of his most famous and bad-tempered quotations Quine contended that "philosophy of science is philosophy enough", meaning that whatever supposed philosophical notion which isn't required in a logical foundation of science is not a legitimate philosophical notion and it does not deserve our attention. Despite the fact that not many philosophers agree with this standard of philosophical legitimacy today, I intend to argue that this is an untenable standard even for Quine himself. According to it some of his own philosophical efforts are illegitimate. In order to do that I'll show first how his notion of ontological commitment do not respect this standard, and then I'll argue that when Quine realized this fact he simply stopped talk on ontological commitment and substituted it by a notion of ontological indifference.

4.9 Diagramas, Fórmulas e Algo Mais uma reflexão sobre os ingredientes das demonstrações matemáticas a partir do exame de provas por *reductio ad absurdum*

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Resumo. Numa das primeiras peças de filosofia da matemática de que se tem notícia — a saber, uma passagem do diálogo platônico *Mênon*, na qual um escravo sem familiaridade com geometria é exortado a indicar qual linha deve ser levada em consideração quando se pretende duplicar a área de um quadrado dado — os diagramas são considerados elementos importantes para que a solução do problema seja vislumbrada, mas ao mesmo tempo perigosos quando o papel dos mesmos deixa de ser entendido como o de mero auxiliar do processo. Com o passar do tempo, esta postura crítica com relação ao uso de diagramas se consolidou, ao passo que as fórmulas algébricas e outros elementos simbólicos utilizados na aritmética não foi submetida a semelhante exame critico. Atualmente, à luz de novas teorias em filosofia da matemática, a importância dos diagramas enquanto simbolismo tem sido recuperada, e a sua relação com outros expedientes simbólicos — tais como as fórmulas algébricas e os símbolos numéricos — pode ser analisada sob novas perspectivas. Neste trabalho, pretendemos retomar a problemática lançada por Platão, colocando em questão o papel dos diagramas e também dos demais artefatos simbólicos tais como fórmulas e numerais na prática matemática. Paralelamente a isso, tentaremos entender o papel da linguagem (num sentido discursivo, não-formal) na prática matemática. Para atingir este fim, lançaremos mão da análise de provas por *reductio ad absurdum* tanto em geometria quanto em aritmética. Nestas demonstrações, como se sabe, uma suposição inicial é abandonada (em detrimento de sua negação) porque leva a uma contradição. De acordo com interpretações recentes acerca das demonstrações em geometria euclidiana, nas demonstrações por *reductio*, a contradição surge da inter-rrelação entre a dimensão discursiva, por assim dizer, e a dimensão simbólica. Na parte discursiva se estabelece uma determinada propriedade a respeito do que está sendo tratado, e na parte simbólica fica explicitamente constatado a falha em representar tal situação. Isso ocorre, por exemplo, em geometria, quando o raciocínio nos leva a sustentar uma igualdade entre elementos que são representados graficamente como um sendo parte do

outro (o que os impossibilita, na geometria euclidiana, pelo menos, de serem tidos como representando coisas iguais); e em aritmética quando um determinado número é dito possuir uma determinada propriedade que os símbolos utilizados para representá-lo claramente falham em representar. Com a análise de semelhantes casos, esperamos contribuir para, por um lado, entender melhor o papel da linguagem simbólica (nas suas diferentes formas) nas demonstrações matemáticas; e, por outro lado, investigar o que resta ainda de genuinamente matemático para além da linguagem simbólica nas práticas matemáticas em geral.

4.10 Meaning and composition as procedures

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Abstract. Consider the following version of Frege's puzzle:
A = A is not informative.
A = B is informative.
1. A and B have the same extension. (Assumption)
2. Extensions = meanings. (Assumption)
3. We know the meanings of our expressions. (Assumption)
4. A and B have the same meaning.
5. We know that 'A' and 'B' have the same meanings.(and the meaning of '=')
6. If we know that 'A' and 'B' have the same meanings, then we know that 'A = B' is true. (Granted)
7. We know that 'A = B' is true.
8. However, we don't know that 'A = B' is true.
REDUCTIO
Typically, premise 2 is rejected in light of Frege's puzzle. Note however, that 1 and 3 are assumed as well. Suppose that language is the vehicle for knowledge and something we are, not something we know. In that case, competency with language doesn't provide knowledge of meanings, and if the speaker doesn't know that A = B, then she doesn't know something about the meanings of her terms. You might wonder how she manages to use language, if the she doesn't know the meanings of her expressions. However, that assumes that knowledge of language is required for proper use of language. But this is exactly what is being rejected and the reason for positing intensions, or senses, in addition to extensions. There are issues other than cognitive significance to worry about. That response to the puzzle doesn't in itself explain variations in the truth conditions after substitution of co-extensional expressions in intensional contexts. However, it might suggest alternative strategies for dealing with that issue. Frege (1892) argued that both (1) "the author of Waverley" and (2) "the author of Ivanhoe" denoted Walter Scott. Thus compositionality meant that they were substitutable salva veritate. But in the attitude cases such as (3) Maria believes that Walter Scott is the author of Waverley and (4) Maria believes that Walter Scott is the author of Ivanhoe this would lead to violating compositionality because such sentences would possibly denote distinct tuth-values. Hence Frege invented the reference shift, which solved the problem but at the cost of contextualism. The

mission of this presentation is to argue that the denotation cannot be dependent on context since it is what the sense constructed and the sense is absolutely independent of context. The analysis is based on Transparent Intensional Logic. The payoff is considerable: in TIL we use the term "denotation" for the object (if any) that is constructed by the sense. The term "reference" is used for the object (if any) that happens to be the value of the intension denoted by an empirical expression in the actual world. Denotation is an exactly defined object a priori constructed by the sense. Reference is (possible) contingent value of a denotation in the actual world. For the purposes of natural-language analysis, we are assuming the following base of ground types, which are part of the ontological commitments of TIL:
ο: the set of truth-values $\{T, F\}$;
ι: the set of individuals (constant universe of discourse);
τ : the set of real numbers (doubling as temporal continuum);
ω: the set of logically possible worlds (logical space).
The logical core of TIL is its notion of *construction* and its *type hierarchy*, which divides into a ramified type theory and a simple type theory. Meaning of any expression is a construction. As such, the meaning is independent of any context. What is dependent on a context is the way the meaning is handled in the context. This is not intended to be a snapshot of what Frege was thinking. What it does do is make it clear that in (1), (2), (3) and (4) only the reference happens to be the same.

4.11 Intensions as Algorithmic Semantics

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Abstract. The terms in the natural language are typically used to point to/designate objects in the world. For example, the term 'Venus' in the sentence "Venus does not have an atmosphere" designates one of the planets in our solar system. However, an important characteristic of natural languages is precisely the fact that they are capable of speaking not only of the objects in the world, but also of language itself, as well as the speakers of the language. Here we have some examples:

(1) "The sentence 'Shakespeare were the greatest writer of all time' has a grammatical error"

(2) "German is a beautiful language"

(3) "John believes that the earth is flat"

Traditionally, the semantic of formal languages intended to represent aspects of natural language are purely denotational, that is, all terms and expressions of the formal language designate an object or a truth value. Even the modal approaches used to deal with intensional contexts are base on possible world semantics where the terms have a designation in each possible world.

The starting point for the development of the proposed notion of algorithmic semantics is the observation that it doesn't make sense, in the example (1) above, to consider the term 'Shakespeare were the greatest writer of all time' as a name for the corresponding sentence or proposition that would exist in the domain of objects. Rather, we should realize that the predicate in sentence (1) applies to linguistic expressions, and that the term 'Shakespeare were the greatest writer of all time' is exactly the linguistic expression to which the predicate is being applied. In other words, the term that appears in this sentence does not *refer* to an object, but is itself *the object* which is being talked about. Next, we observe that linguistic objects do not have the same nature as the objects of the world: their properties are not assessed through perceptual inspection, but are defined by the conventional rules of the language. These two observations give rise to the idea of a semantic value that is produced by the execution of an algorithm which verifies a set of rules.

More specifically, we propose a mathematical model that represents these aspects of language in a suitable way according to the aspects we discussed above. We propose a first-order logic where some predicates do not have an extension, but are associated to an algorithm that is used to evaluate it. As a simple example, the language of such logic have expressions like $WFF('\forall xP(x)')$ and an interpretation for the language associates to the predicate symbol WFF an algorithm that check whether the term $'\forall xP(x)'$ is a well-formed formula. The algorithm is not intended to be the designation of the relation symbol WFF, but its intention, and is used to evaluate it.

Now, despite the ingenuous appearance of the example adopted in the discussion above, the notion of algorithmic semantics opens the way to a number of interesting possibilities: i) quantification over terms; ii) an interpreter inside the language, so that the language may play the role of its own metalanguage, which is a feature of the natural languages, iii) an alternative semantics for the mathematical discourse, iv) predicates which involve both syntactical manipulations as well as consulting the properties of the objects designated by the terms, v) proof-theoretic semantics as a special case of algorithm semantics, and so on.

4.12 Logics with Relational Fixed-Point Operators

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Abstract. Descriptive Complexity is a field of Finite Model Theory, which is interested in characterizing computational complexity classes in terms of the logical resources that are required to express all the problems belonging to the class. The seminal result in the area is the celebrated Fagin's Theorem, which proves that the class NP is captured by the existencial fragment of second order logic [Fagin 1974]. On the other side of the spectrum, we have the well known fact that first-order logic is not sufficiently expressive to define even such simple problems as graph connectivity. The introduction of fixed-point operators is a standard technique to increase the expressive power of a logic in a controlled way. Indeed, [Immerman 1982] and [Vardi 1982] prove that first-order logic with the least fixed-point operator, denoted FO(LFP), is able to capture the class P over the set of ordered strutures. In an extension of this work, [Abiteboul; Vardi and Vianu 1997] introduces the notion of non-deterministic fixed-points and proves that the first-order logic which such operators captures the class NP. In this work, we follow a different approach and define the notion of the fixed-point of an arbitrary relation R. The basic idea is that X is a fixed-point of the relation R in case the pair (X, X) belongs to R. We then introduce the notion of the inductive inflationary fixed-point of the relation R and the associated operator RIFP. We denote by RIFP(FO) the fragment of the first-order logic with the inflationary relational fixed-point operator RIFP, with the restriction that this operator can be applied at most once, as the most external element of the formula. Our main result is the proof that the logic RIFP(FO) captures the class NP.

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4.13 Up the hill: on the notion of information in logics based on the four-valued bilattice

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Abstract. The present paper aims to explore the role of information in logics based on the classical bilattice FOUR $= \langle \mathbf{4}, \leq_t, \leq_i, -_t, -_i \rangle$ [Fitting 2006], where $\mathbf{4} = \{\mathbf{f}, \bot, \top, \mathbf{t}\}$. The information order $\leq_i$ is usually understood as the order that goes from "lack of information" to "excessive information". This interpretation results in situations in which a given sentence is neither true nor false, or both truth and false. I claim, instead, that the usual sense of informational order is better explained by cognitive attitudes of a society of agents concerning the acceptance and the rejection of a given piece of information. In that sense, the usual four truth-values may be defined from the primitive notion of cognitive attitudes in the following way. Given a society of agents consulted about a sentence φ, the values of φ are:

$\mathbf{f}$ — *if there is no agent who does not reject or accepts* φ;
$\bot$ — *if there is no agent who accepts or rejects* φ;
$\top$ — *if there is no agent who does not reject or does not accept* φ;
$\mathbf{t}$ — *if there is no agent who does not accepts or rejects* φ.

The definition of entailment relation in Dunn-Belnap's logic is based on the logical matrix that takes the bifilter $\mathcal{F}=\{\top, \mathbf{t}\}$ of $FOUR$ as the set of designated values. Nonetheless, other entailment relations can also be defined if one changes the set of designated values [Marcos 2011].

One particularly well motivated non-standard matrix is defined considering the acceptance and the rejection of a given piece of information by a given society of agents: the matrix $\mathcal{M} = \langle \mathbf{4}, \mathsf{Y}, \mathsf{N}, \mathcal{O} \rangle$. Such $\mathcal{M}$ is a generalized quasi-matrix [Shramko and Wansing 2011], that takes two sets of distinguished values, one for acceptance $\mathsf{Y} = \{\top, \mathbf{t}\}$, and the other one for rejection $\mathsf{N} = \{\mathbf{f}, \top\}$. The set $\mathcal{O}$ is the set of operations interpreting the logical language.

Let ⅄ be the complement of Y and И be the complement of N relative to $\mathbf{4}$. Given a set of formulas Γ, an agent s and $\mathsf{A} = \{\mathsf{Y}, ⅄, \mathsf{N}, И\}$, $\mathsf{A}s{:}\Gamma$ means s *holds the cognitive attitude* A *concern the sentences of* Γ. The associated entailment relation, called B-entailment, is defined in the following way:

$\frac{\Psi}{\Gamma}\Big|\frac{\Delta}{\Phi}$ *if there is no agent* s *such that* $\mathsf{Y}s{:}\Gamma$ *and* ⅄$s{:}\Delta$ *and* $\mathsf{N}s{:}\Phi$ *and* И$s{:}\Psi$.

(that means, there is no agent s such that s accepts all sentences of Γ and s does not accept all sentences of Δ and s rejects all sentences of Φ and s does not reject sentences of Ψ).

This notion of entailment expresses many different kinds of reasoning concerning acceptance and rejection. On the present contribution, I will explore statements of the form $\frac{\beta}{\alpha}\Big|\frac{\cdot}{\cdot}$ and $\frac{\cdot}{\cdot}\Big|\frac{\beta}{\alpha}$, expressing respectively that '*there is no agent that accepts α and does not reject β*' and by '*there is no agent that rejects α and does not accept β*'. These two statements correspond to two not so obvious reasoning forms concerning the information order of the bilattice $FOUR$.

I note that a logic based on $\mathcal{M}$ such that the operators of $\mathcal{O} = \{\oplus, \otimes, -_i\}$ correspond to meet, join, and inversion of $\leq_i$, is isomorphic to a logic whose operations are $\{\vee, \wedge, -_t\}$ that correspond to meet, join, and inversion of $\leq_t$. Conceptually, however, the former logic gives us some intuitions about how a Dunn-Belnap system could deal with information. For example, if we remove **t** or **f** from the set of values the resulting logic will be isomorphic to the Logic of Paradox LP (or to the strong Kleene's Logic). Nonetheless, the remaining values give us something very unusual, since the expected glutty (resp gappy) value turns to be **t** (resp **f**).

Besides understanding what the reasoning based on information order could mean and how it works, I will also introduce a language capable of connecting the information and the logical orders of $FOUR$, namely a logic whose operations are $\mathcal{O} = \{\vee, \wedge, -_t, \oplus, \otimes, -_i, \curvearrowleft\}$, where $\curvearrowleft$, the "quarter turn" [Ruet 1996] is defined by letting $\curvearrowleft\bot$:=**t**, $\curvearrowleft$**f**:=$\bot$, $\curvearrowleft$**t**:=$\top$, and $\curvearrowleft\top$:=**f**.

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4.14 Conditional antinomies

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Abstract. It is a well-known fact that conditionals, and especially counterfactual conditionals, are a source of paradoxes. In this connection one may quote Lewis Carrolls' barbershop paradox, the Bizet-Verdi paradox formulated by Quine and what Jon Elster calls the paradox of the "Unimportance of the Inevitable" (see [Elster 1978]). Other less known paradoxes could also be recalled, as the puzzle of what one might call "counterlinguistic" conditionals ("if the tail of a dog were calle paw, how many paws would have a dog?"). More recently, J. L. Pollock discovered a possible antinomy in the analysis of counterfactuals in terms of comparative similarity of possible worlds (see [Pollock 1981])

A problem which has been completely neglected in the literature about the paradoxes concerns the Liar - like antinomies in which the basic is a non-material conditional. There at least two prima facie variants of the relevant statement:

(L1) If this conditional were true it would be false

(L2) If this conditional were false it would be true

The interest of (L1) and (L2) is suggested by the fact that the standard Liar antinomy ("this statement is false") arises thanks to a conditional argument of the same form of (L1) and (L2) (if this statement were true it would be false and if this statement were false it would be true).

A parallel discussion should be devoted to the so-called "truth-teller" variants of the above antinomies, which however in the more obvious formulation appear to be tautological:

(#) If this conditional were true it would be true

(##) If this conditional were false it would be false

One can also imagine that there is also an Epimenides-like paradox which might be formulated as

(°) Given that all Cretans lie, if I were a Cretan I would lie. or also, more properly:

(°°) Epimenides the Cretan says that, for every x, if a person x were a Cretan, x would be a liar.

The paper aims to giving evidence that the treatment of the paradoxes depends on the truth-conditions accepted for conditionals, which in their turn depend on some presupposed semantical theory. The two dominant semantical theories about conditionals are the Chisholm-Goodman-Reichenbach theory (according to which the truth of a conditional depends on the consequence nexus between the clauses) and the Stalnaker-Lewis theory (according two which the truth of a conditional depends on the inspection of possible worlds). The proposed analysis aims to examining the differences in the treatment of the paradoxical statements which depend on the choice of the background semantical theory.

An interesting question concerning the background semantical theory is that it is plausible to maintain that the truth values for conditionals may be indeterminate. As J.L. Mackie remarks in [Mackie 1973], a variant of the Liar is

(*) This statement, standardly construed, is indeterminate So an extended discussion of the above problem has to analyze the following two conditionals

(**) If this conditional were indeterminate, it would be true

(***) If this conditional were true, it would be indeterminate.

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4.15 Abstraction and Exensionality as conflicting principles at the origin of set- theoretic paradoxes

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Abstract. In this talk I will focus on some structural features of the most common paradoxes of set theory (Russell, Burali Forti, Cantor, Richard). My contention is that the outcomes of Neo-Fregean logicist theories may shed new light on the generation of inconsistencies within naïve set theory.

In the first part of the talk I will introduce the notion of extension of a concept as a representative and paradigmatic logical object (Ruffino [2000], Ruffino [2003]). Through the paradox of the concept "horse" (Frege [1988]) I will show that there is a logico-syntactic hierarchy among Frege's logical objects (which contemplates as minimum extensions, truth values, and natural numbers) and I will maintain that this conception of extension is necessary to "represent" in a formal language not only numbers and truth values, but also relations between concepts; for the this reason extensions are required by logic as Frege conceived it.

In the second part of the talk I will carry out a preliminary analysis of Frege's way of introducing extensions, namely the Basic Law V as stated in the *Grundgesetze der Arithmetik*. This axiom has been historically labeled as "infamous" due to its inconsistency. In order to do without it, Crispin Wright, in his famous 1983 book Frege's Conception of Numbers as Objects developed formal proofs to derive Peano-Dedekind's axioms from what has been called Hume's Principle. On the basis of Ruffino [2003], I will argue that Wright's strategy – which is inspired by Frege's early attempt to define natural numbers in Frege [1980] – is unacceptable in the framework of Frege's conception of Logic, since it gives up extensions at all. I will then draw the conclusion that a Neo-Fregean theory of arithmetic should be set theoretical rather than number theoretical; in other words the Neo-Fregean philosopher cannot escape the Basic Law V.

In the third part of the talk I will analyse the inconsistency of the Basic Law V and eventually the engendering of Russell's paradox. By referring to Russell [1906], Dummett [1963], Dummett [1991], Shapiro [2003], Luna Taylor [2010] I will highlight in what sense the common feature of many set theoretical paradoxes is "indefinite extensibility". I will go through some at-

tempts of ”repairing” the Basic Law V and I will show to what extent they work and what they left unsolved.

In the fourth part of the talk I will try to spell out what requirements a revised Basic Law V should meet to capture the notion of indefinite extensibility. I will argue that the paradoxes arise due to a twofold character of the axiom (one semantic and the other syntactic) which leads to an internal conflict: on the one hand it works as a principle of extensionality, i.e. it allows extensions as completely defined objects; on the other hand it works as a principle of abstraction, i.e. it allows the introduction of infinitely many new objects by a re-conceptualisation of the logical equivalence between propositional functions. I will point out that though the syntactic form of each character of the axiom is the same, the semantical import is different.

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4.16 Displaying Dynamic Logics

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Abstract.
Display calculi. Nuel Belnap introduced the first display calculus, which he calls *Display Logic* [Belnap 1982], as a sequent system augmenting and refining Gentzen's basic observations on structural rules. Belnap's refinement is based on the introduction of a special syntax for the constituents of each sequent. Indeed, his calculus treats sequents $X \vdash Y$ where X and Y are so-called *structures*, i.e. syntactic objects inductively defined from formulas using an array of special connectives. Belnap's basic idea is that, in the standard Gentzen formulation, the comma symbol ',' separating formulas in the precedent and in the succedent of sequents can be recognized as a metalinguistic connective, of which the structural rules define the behaviour.

Belnap took this idea further by admitting not only the comma, but also several other connectives to keep formulas together in a structure, and called them *structural connectives*. Just like the comma in standard Gentzen sequents is interpreted contextually (that is, as conjunction when occurring on the left-hand side and as disjunction when occurring on the right-hand side), each structural connective typically corresponds to a pair of logical connectives, and is interpreted as one or the other of them contextually. Structural connectives maintain relations with one another, the most fundamental of which take the form of adjunctions and residuations. These relations make it possible for the calculus to enjoy the powerful property which gives it its name, namely, the *display property*.

Belnap's cut elimination metatheorem. In [Belnap 1982], a meta-theorem is proven, which gives a set of sufficient conditions in order for a sequent calculus to enjoy cut-elimination and subformula property. This meta-theorem captures the essentials of the Gentzen-style cut-elimination procedure, and is the main technical motivation for the design of Display Logic. The sufficient conditions in Belnap's meta-theorem are relatively easy to check, since most of them are verified by inspection on the shape of the rules. When Belnap's metatheorem can be applied, it provides a much smoother and more modular route to cut-elimination than the Gentzen-style proofs. Moreover, cut-elimination Belnap-style has the important advantage of being preserved under the addition of structural rules and introduction rules for new logical

connectives,[1] whereas a Gentzen-style cut-elimination proof for the modified system cannot be deduced from the old one, but must be proved from scratch. In a slogan, we could say that Belnap-style cut-elimination is to ordinary cut-elimination what canonicity is to completeness: indeed, canonicity provides a *uniform strategy* to achieve completeness. In the same way, the conditions required by Belnap's meta-theorem ensure that *one and the same* given set of transformation steps is enough to achieve cut elimination for any system satisfying them.[2] Various refinements and extensions of the original notion of display calculi exist in the literature, e.g. the proper display calculi in [Wansing 1998, Section 4.2] for the former, and [Belnap 1990] for the latter.

Contribution. The proposed contribution aims at reporting on the recent advances of a line of research [Greco et al. 2014, Frittella et al. 2015b;a;c; 2014, Frittella and Greco 2014, Greco; Kurz; Palmigiano 2013] aimed at 'displaying dynamic logics'. The new advancements make it possible to overcome the hurdles specific to the settings of Baltag-Moss-Solecki's Dynamic Epistemic Logic (DEL) [Baltag; Moss and Solecki 1999], Propositional Dynamic Logic (PDL) [Harel; Kozen; Tiuryn 2000], and monotone modal logic [Chellas 1980, Hansen 2003].

Methodology. The solutions to the specific technical difficulties of each logical system mentioned above require generalising Belnap's meta-theorem along different dimensions. Specifically, key to displaying DEL and PDL is the introduction of a *multi-type environment* for display calculi. This environment makes it possible to treat the parameters (actions, agents) of the modal connectives as terms in their own right. The difficulties in the treatment of the preconditions to the applicability of certain rules are dealt with by a suitable expansion of the language. Moreover, the display property is guaranteed by the introduction of certain structural connectives, referred to as *virtual adjoints* in [Frittella et al. 2015c], since they do not have any semantic interpretation. The price to pay to this language expansion is that one must prove separately that the resulting calculus is a conservative extension of the original logic. This was achieved in the case of the typed calculus for DEL with a relatively concise and smooth proof. The analogous proof for PDL is still an open problem.

The specific difficulty posed by monotone modal logic is the fact that its axiomatisation excludes the existence of the adjoints of the modal connectives.

[1] Provided the rules in question verify certain conditions which we do not discuss here.

[2] The relationship between canonicity and Belnap-style cut-elimination is in fact more than a mere analogy, see [Kracht 1996], and more recently [Greco et al. 2014].

Rather than via virtual adjoints, the solution to this problem has been given in terms of a generalisation of Belnap's meta-theorem for calculi which do not enjoy the display property. Specifically, instead of it, the calculi are required to satisfy (a slight relaxation of) Sambin-Battilotti-Faggian's *visibility property* [Battilotti; Faggian and Sambin 2000].

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4.17 A consciência emerge da matéria? O fisicalismo real de Galen Strawson vs. o pós-fundamentalismo de Robert Hanna

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Resumo. A teoria da mente essencialmente corporificada proposta por Hanna e Maiese no livro intitulado "*Embodied Minds in Action*" (2009), pretende ser uma terceira via efetiva entre o dualismo cartesiano e o eliminativismo, liberta das instabilidades teóricas das diversas propostas disponíveis, tais como o dualismo de propriedades, o funcionalismo e, em especial, o monismo anômalo de Davidson. A experiência consciente tem um lugar meritório na abordagem de Hanna e Maiese; de fato, eles iniciam a investigação tendo sua existência como ponto de partida teórico. Eles buscam desenvolver, em termos lassos, um tipo especial de fisicalismo não redutivo, a partir de uma metafísica modal que envolve a teoria dos sistemas dinâmicos e uma versão renovada do hilemorfismo aristotélico.

Uma das maiores dificuldades que qualquer posição não reducionista em filosofia da mente precisa enfrentar é mostrar como se dão as relações entre as propriedades da experiência consciente e as propriedades físicas. A estratégia comumente usada para responder a essa dificuldade é mostrar que as propriedades mentais "surgem" das propriedades físicas, seguindo a intuição educada que nos diz que a matéria organizada apropriadamente em cérebros ou, talvez mais corretamente, em corpos vivos, "faz surgir" a experiência consciente.

Hanna e Maiese pretendem oferecer um corpus teórico apropriado para essa intuição. Para tanto, eles precisam caracterizar o que significa dizer que o físico "faz surgir" o mental, apresentando uma teria da emergência dinâmica das propriedades mentais em uma corporificação adequada, vinculada a uma teoria mais geral, a teoria da corporificação essencial da consciência.

Em um famoso artigo de 1996, intitulado "Realistic Monism: Why Physicalism Entails Panpsychism", Galen Strawson defende, entre outras coisas, a tese de que qualquer teoria da emergência de fenômenos experienciais a partir de fenômenos não experienciais irá fracassar, devido ao fato de que tais teorias não são capazes de explicar "em virtude de que" a emergência ocorre.

Sem tal explicação qualquer teoria da emergência do mental a partir do físico é sempre emergência bruta, ou seja, equivalente a um milagre. No trabalho a ser apresentado, mostrarei que a crítica de G. Strawson pode ser estendida à emergência dinâmica, a despeito do fato de que a teoria da emergência dinâmica proposta por Hanna e Maiese ser teoricamente mais complexa e profunda do que as teorias da emergência criticadas por G. Strawson.

A metafisica modal dos sistemas dinâmicos proposta por Hanna e Maiese está estreitamente vinculada com a visão pós-fundamentalista do mundo, explicitada na tese da emergência dinâmica, segundo a qual o mundo não é nem fundamentalmente físico nem fundamentalmente mental, mas uma totalidade de forças dinâmico-causais. A adoção do materialismo pós-fundamentalista nos dá o conforto necessário para afirmar que a matéria pode, dada uma situação correta, estar essencialmente vinculada, ou seja, fundida ao mental. Tal situação se dá quando as propriedades mentais emergem dinamicamente da complexidade física e biológica, constituindo um organismo essencialmente físico e mental. A fusão de propriedade físico-mental é definida por Hanna e Maiese nos seguintes termos:

1. Sob uma corporificação E, um evento ou substância física X tem determinadas propriedades mentais fundamentais M^1, M^2, M^3, etc.

2. Sob a mesma corporificação E, X também tem determinadas propriedades físicas fundamentais, não idênticas ou distintas, P^1, P^2, P^3, etc.

3. Para toda Mi há uma correlação um a um com a Pi correspondente.

4. Os membros de cada par Mi-Pi, correlacionados 1 a 1, são necessariamente co-extensivos.

5. Os membros de cada par Mi-Pi, correlacionados 1 a 1, não são logicamente necessariamente co-extensivos.

6. Os membros de cada par Mi-Pi, correlacionados 1 a 1, são propriedades estruturais intrínsecas ou mutuamente inerentes de X.

7. X é um organismo vivo apropriadamente complexo. (Hanna e Maiese, p 354)

A fusão físico-mental é exemplificada, dentre todos os mundos possíveis, apenas em mundos constituídos por sistemas dinâmicos. Isso se deve a (5), que diz que as propriedades mentais e propriedades físicas são necessariamente co-extensivas metafisicamente, e não logicamente. Este argumento metafísico-modal eliminaria de plano a tanto a identidade de propriedades, por exigir a necessidade lógica da co-extensão de propriedades, quanto a superveniência lógica, que requer relações lógicas de suficiência (Hanna e Maiese, p. 328).

Mostrarei também que a tese da emergência dinâmica advoga algo bem próximo da estratégia, criticada por G. Strawson, de defender a possibilidade da emergência do mental a partir das propriedades proto experienciais do físico. Na terminologia de G. Strawson, o não experiencial é proto experiencial se é intrinsecamente ajustado a constituir, em certas circunstâncias, fenômenos experienciais. O mundo constituído de sistemas dinâmicos de Hanna e Maiese seria proto-experiencial do ponto de vista de G. Strawson. No entanto, se partirmos do ponto de vista pós-fundamentalista de Hanna e Maiese, a crítica de G. Strawson à emergência do mental a partir do físico seria restrita à emergência como superveniência, e assim, subalterna à visão mecânica e estratificada do mundo.

Meu objetivo geral é mostrar que a crítica de G. Strawson à emergência como superveniência pode ser estendida à emergência dinâmica de Hanna e Maiese. Segundo os últimos, a emergência dinâmica de propriedades físico-mentais se dá através da fusão de propriedades físicas e de propriedades mentais em uma corporificação específica, constituindo um novo tipo de sistema dinâmico, o organismo vivo animal. Mas a fusão de propriedade físico-mental que resulta da emergência dinâmica se dá em virtude de quê? Hanna e Maiese entendem que são as propriedades mentais que emergem dinamicamente, dada uma corporificação adequada. Se perguntarmos em virtude de que as propriedades mentais emergem das propriedades físicas, teríamos de responder, se quisermos seguir Hanna e Maiese, que as propriedades mentais emergem dinamicamente em uma determinada corporificação, dadas as circunstâncias adequadas. No entanto, já que as propriedades mentais são propriedades novas e inauditas, então tais circunstâncias precisam ser fundamentalmente físicas, fazendo com que a posição pós-fundamentalista de Hanna e Maiese se colapse no fundamentalismo que eles rejeitam.

4.18 Uma contribuição ao tema da consciência corporificada

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Resumo. O objetivo deste trabalho é mostrar o lugar do corpo e suas implicações filosóficas para a cognição num modelo alternativo de consciência que se situa além do modelo clássico de consciência. Com uma possível hipótese para um sujeito pós cartesiano mostrarei a natureza da consciência corporificada com suas propriedades e como esta pode oferecer respostas para a ação e relação mente-corpo, assim como sua ação no mundo. Indagarei como a modelo da consciência corporificada e suas teses podem contribuir para as pesquisas em filosofia da mente ao analisar a estrutura disposicional de estados mentais, como crenças, razões de agir e desejos, salientando sua importância tanto em sua descrição como em sua atuação no mundo. Seguindo a direção dos autores Hanna e Maiese tentarei responder: O que são criaturas com corpos como nós, neurologicamente corporificadas com uma consciência enraizada no mundo, orientada de forma dinâmica e organizada por estruturas espaciais e temporais. Para tanto, explorarei primeiramente o modelo epistêmico para a mente de Hanna e Maiese (H&M) contido em "*Embodiment Mind in Action*" (2009) e suas teses principais: (i) A corporificação essencial da consciência. A consciência humana é corporificada ou a consciência humana é incorporada, uma encarnação neurobiológica de todos os sistemas e órgãos vitais do homem, homem animal vivo e consciente (cérebro, sistema nervoso, sistema límbico, cardiovascular) até os limites da pele e com uma extensão expandida aos domínios do ambiente. (ii) Tese da consciência profunda. Os estados mentais são conscientes e penetram em todos os aspectos da nossa vida mental. Para os autores citados, não há pensamentos inconscientes ou sub-pessoais, todos os estados e processos mentais pressupõem subjetividade sensório-motora e esse é o modo primitivo de consciência corporal que fornece a base para significado de si mesmo e a capacidade de uma perspectiva subjetiva e objetiva. Além dessas duas grandes teses, H&M exploram uma outra tese que dá ênfase ao corpo e suas difíceis relações concientes com o mundo. (iii).Tese da animalidade física e mental. Esta é uma versão renovada do hylemofismo aristotélico e sob este aspecto sugere uma oposição ao dualismo cartesiano. Para os autores, as propriedades físicas e mentais são fusionadas, são sistemas dinâmicos, de forças dinâmicas e causais próprias ao organismo vivo, forças que impulsionam um corpo neurologicamente complexo que se movimenta e age. Em sequência a essas teses, Maiese, num

outro livro, "*Embodiment, Emotion and cognition*" (2011) explora estes sistemas dinâmicos, cujo significado é, obviamente, materialista e pós fundamentalista e os adiciona a um sujeito corporificado com raízes biológicas e naturais. De acordo com alguns autores, como Thompson e Varella, experiências pessoais partem de um ponto de vista centrado, cujo sujeito não é conceitual ou uma experiência auto reflexiva, apenas neurobiologicamente centrado. No entanto, o sentido do sujeito e da consciência, por definição, é imanente e reflexivo e esta corporificação essencial terá necessariamente um conatus sensorimotor, afectus subjetivo na definição de uma consciência corporificada. Este modelo de fusão considera as conexões entre o desejo, emoção, ação e experiência corporal. Para os autores, há portanto, uma reflexividade imanente, intransitiva, não intrínseca, mas relacional. Um dos pontos explorado é a relação espaço-tempo, onde a espacialidade e o sentido do tempo têm estrutura e propriedades intrínsecas (posição do corpo no espaço e percepção do tempo) num sistema específico: diacrónico e sincrónico. Maiese para caracterizar a estrutura da consciência mantém o modelo bidimensional de Searle - unidade horizontal e vertical, diacrónica e sincrónica. Estas propriedades intrínsecas e relacionais são acidentalmente externas e necessariamente internas, assimétricas, mas globalmente orientadas no espaço e tomadas no tempo. Em H&M são chamadas 'propriedades estruturais intrínsecas'. Os benefícios destas estruturas espaço-temporais é que elas mantém relações com o meio ambiente. Portanto, o ponto de origem da consciência é um corpo com uma continuidade de experiências perceptivas numa dimensão de engajamento no mundo. Acrescentarei a esta exploração meu interesse pessoal e minha hipótese de trabalho: que essa consciência corporificada assim estruturada não é apenas um fenômeno no mundo, dado numa experiência fenomenal, a importância deste corpo é um significado e sentido de corpos entre outros corpos. Esta plasticidade do corpo que é parte do conceito de "mundo circundante" ou "*Umwelt*" é necessária para o conhecimento, mas por si só não é suficiente, porque além de sua estrutura natural necessita de uma estrutura disposicional, como estados mentais, crenças, razões para a ação expressas por frases verdadeiras ou falsas. Ao supor essa estrutura e abordá-la, minha questão sobre a consciência corporificada que tem o corpo como base, busca nele um suporte para as relações de significado de sujeito/sujeitos no mundo, (Davidson), para além de suas manifestações e de sua ação no mundo seja como descrição neurofisiológica e neurobiológica ou ainda como descrição fenomenológica ou então, como uma resposta naturalista primitiva.

4.19 A importância da esfera transcendental no pensamento lógico e metafísico de Wittgenstein

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Resumo. Ludwig Wittgenstein, filósofo contemporâneo do século XX, transita seu pensamento desde a filosofia analítica àquilo que o filósofo denomina de místico. Desta forma, a relação entre lógica e metafísica que a princípio parece pouco provável, torna-se um diálogo possível quando as duas esferas se tangenciam por intermédio da natureza dos elementos que solidificam a estrutura tanto da lógica quanto da metafísica. Eis a questão: qual é a natureza desse elemento de sustentação? Transcendental. Wittgenstein percebe uma natureza transcendental tanto na lógica quanto no plano metafísico ou também chamado de místico. A filosofia contemporânea de Wittgenstein transita entre os dois eixos, por vezes aproximando-os, por outras vezes afastando-os. Sabemos que são esferas de diferentes estruturas, ainda que haja um elemento transcendental em ambas. Com a finalidade de uma maior compreensão da complexa filosofia de Wittgenstein, faz-se necessário alcançarmos o entendimento daquilo que o filósofo compreende como "mostrar". O "mostrar" não é do domínio e nem depende da vontade humana. Esta esfera envolve o complexo pensamento filosófico do autor, no que se refere ao entendimento do ético, estético, religioso e lógico. Os três primeiros temas se referem ao mesmo tipo de mostrar, isto é, o "mostrar místico" ou também chamado de "mostrar ético". O "mostrar místico" relaciona-se à atmosfera daquilo que não é fatual e, portanto, do que não possui um conteúdo descritivo. Quanto à lógica, o mostrar é diferenciado. O "mostrar lógico" está relacionado à linguagem e seus respectivos limites. Nesse caso, tangencia aquilo que é da ordem dos fatos. Já que a linguagem expressa os fatos e estes são expressos por intermédio das proposições, a proposição é a expressão linguística dos fatos. O que permite à proposição descrever o fato é a forma lógica comum a ambos. É nesse aspecto que o "mostrar lógico" revela-se na linguagem, através da forma lógica que está ligada à essência do mundo. Seja no "mostrar místico" ou no "mostrar lógico", observamos que os elementos a eles ligados são de aspectos metafísicos. A natureza metafísica é o impedimento para falar a respeito daquilo que se mostra. Embora este seja o assunto o qual abordaremos com maior cuidado, faz-se necessário ressaltarmos a esfera do "dizer" enquanto a dimensão associada aquilo que é factual.

O aspecto dizível não é problemático, haja vista que se refere aos fatos. Tudo que está no mundo é, por conseguinte, um conjunto de fatos. E o que é da ordem dos fatos, possui a possibilidade de ser dito através da linguagem. Em nosso trabalho, abordaremos as duas dimensões "dizer" e "mostrar" enquanto complementares uma da outra e não excludentes. Nosso trabalho objetiva investigar a dimensão do transcendental a partir da compreensão do "mostrar" no pensamento filosófico de Wittgenstein. Pretendemos explanar que o domínio do "mostrar" se subdivide em ético e lógico. Mesmo subdividindo-se, observamos que o ético e o lógico se integram na unidade daquilo que se mostra. O período que abordaremos com mais afinco será a chamada primeira fase (1914-1929) do pensamento de Wittgenstein. Caso seja necessário, levaremos em consideração alguns elementos da fase intermediária de pensamento filosófico do autor. Estas serão as bases e diretrizes para a nossa investigação que não tem como pretensão esgotar o complexo tema.

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4.20 O que é o problema das constantes lógicas?

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Resumo. Tenho como objetivo o esclarecimento do que consiste o problema das constantes lógicas e as principais motivações filosóficas envolvidas neste. Como Tarski [1980] observou, a relevância filosófica do problema justifica-se na importância que a distinção entre termos lógicos e não lógicos tem para o esclarecimento de noções como o conceito de "analítico", ou mesmo, as concepções de"verdade lógica" e "consequência lógica". Um slogan recorrentemente usado é o de que a lógica é a disciplina que tem por objeto de estudo os raciocínios válidos; uma disciplina eminentemente normativa que fornece ferramentas para que possamos avaliar a validade de nossos argumentos independente de seu âmbito de aplicação. Por sua vez, este empreendimento supostamente depende da exposição das estruturas lógicas subjacentes aos argumentos: aquilo que denominamos de sua forma lógica. Na prática mais comum, as formas válidas de argumentos são determinadas levando-se em consideração uma classe privilegiada de expressões, tais como, "não", "e", "ou", "se..., então ...", "todo" e "existe". Até então, todo o empreendimento filosófico sobre o assunto tem girado em torno da questão de fornecer critérios que possam demarcar o território de expressões que seriam relevantes para os propósitos da lógica. No entanto, o que se torna curioso é que a variedade de critérios formulados apontam em direções diferentes e, teoricamente, são incompatíveis uns com os outros. Além disso, devemos acrescentar o fato de que o desenvolvimento da lógica tem incluído em diversos sistemas formais expressões como o símbolo para identidade, símbolos para operadores modais e para a relação de "pertencer", usada na teoria dos conjuntos, carecem de um critério de demarcação que justifique sua presença no grupo de noções lógicas. Segundo Gómez-Torrente [2002], podemos constatar que grande parte dos critérios de demarcação são baseados em intuições semânticas, epistemológicas e matemáticas.

Um modo de distinguir as expressões que são candidatas a constantes lógicas é apelar para suas propriedades inferenciais: conta como lógica toda a expressão que é caracterizada unicamente por um conjunto de regras inferenciais de introdução e eliminação. Há, pelo menos, dois modos de encarar

este tipo de caracterização. Por um lado, pode-se tomar as regras de inferência como fixadoras do valor semântico de uma constante. Por outro, tais regras fixam o sentido de uma constante. Ver Hacking [1979]. Para outros autores, as regras de inferência determinam o sentido de uma constante lógica. Gentzen (ver Gentzen [1934]), por exemplo, afirma que as regras de introdução fornecem a definição da constante lógica, ao passo que as regras de eliminação podem ser entendidascomo consequências dessas definições.

Quine defendeu a ideia de que critérios gramaticais são adequados para o trabalho de distinguir constantes lógicas de expressões não-lógicas. Grosso modo, sua proposta é a de que constantes lógicas são as partículas gramaticais que formam sentenças complexas a partir da combinação de sentenças atômicas que, por sua vez, são formadas por expressões simples que contariam como termos não lógicos. (ver Quine [1980] e Quine [1986]). A proposta de Alfred Tarski certamente está entre as mais célebres tentativas de solução para o problema das constantes lógicas. Em "What are logical notions?", ver Tarski [1980], Tarski propõe que uma noção conta como lógica caso seja invariante sob transformações um-a-um de uma classe de objetos dentro de si mesma. Embora possamos enumerar uma classe de critérios que permitam traçar as fronteiras entre o que devemos considerar uma constante lógica,penso que devemos esclarecer o que de fato está em jogo. É possível uma formulação precisa do que consiste o problema das constantes lógicas? Podemos fornecer um critério de demarcação unívoco para as expressões que fazem parte dos variados sistemas lógicos existentes? Teorias satisfatórias acerca de constantes lógicas devem estar baseadas em princípios semânticos, epistemológicos, ou de que outro tipo? Minha proposta é de que devemos mudar a ótica sob a qual tem se visto o problema.

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4.21 On Slingshots and Plural Logic

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Abstract. So-called "slingshot arguments" are a family of arguments underling the fregean view that if sentences have reference at all, their references are their truth-values. This family of arguments receives the name of "Slingshot Arguments" in [Barwise and Perry 1981] due to the minimal machinery used and its cogent consequence. Usually seen as a kind of collapsing argument, since once you suppose that there are some items that are references of sentences (as facts or situations, for example), these items collapse into just two items: The True and The False. Frege's liberal notion of proper names states that the reference of a proper name is the unique object for which it stands. Among the sort of things Frege recognizes as proper names are "definite description" and "sentence". A definite description as "The author of *Moby Dick*" is a proper name that refers to a definite object. In a similar way a sentence is a proper name that refers to one of the objects The True or The False.

Russell questions Frege's interpretation of definite description as singular terms. Russell holds that definite descriptions are meaningless without context and that they perform as a quantifiers rather then as singular terms. In what is dubbed Gödel's Slingshot, Kurt Gödel maintains (see [Gödel 1944]) that by using Russell's interpretation of definite descriptions, it is possible to avoid Frege's argument to the conclusion that all true sentences denote The True and all false ones denote The False.

Gödel's Slingshot imposes the challenge that for those who want a Russellian theory of facts must drop at least one of two assumption: compositionality or that definite descriptions are singular terms.

Considering the new developments on the field of Plural Logics in the book "Plural Logic" [Oliver and Smiley 2013] we shall investigate if the Slingshot argument can also be recovered in a context of plural logic when we change the concept of terms to plural terms. We propose a slingshot argument with regard to plural logics. Although Oliver and Smiley agree with Frege that definite descriptions are a kind of proper name, they do not agree with him in considering sentences to be proper names as well. If we are right, the above argument force the authors of *Plural Logic* to put sentences "under the same umbrella" of proper names. A gödelian solution would propose to use some

contextual elimination of plural terms or drop the idea of compositionality. At the end we'll make a comment on why Oliver and Smiley reject completely the strategy of eliminating contextually a plural term.

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4.22 Logic, culture, and rationality

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Abstract. Many authors recognize the existence of a crisis in today's philosophical rationality. An appeal to a scientific model of reason is unlikely to revert such diagnosis: not only have different branches of scientific enquiry quite autonomous sets of methods, conceptual resources, and standards of evidence evaluation, but it usually comes from Philosophy any initiative to state an "underlying order" to be worked out in the form of "rational reconstructions" of scientific reason, invariably resulting in a highly artificial and disputable kind of theorizing – in which scientists themselves seldom manifest any interest. Philosophy may, however, look forward to the development of technical and conceptual tools in order to achieve some level of rational unity. Such was the spirit of the initial growth of modern logic and the establishment of analytical philosophy. However, both "classical logic" and "linguistic analysis" failed to remain as paradigms of philosophical rationality. The unhistorical turn of mind that characterizes the efforts of their leading authors also contributed, to a significant measure, to undercover the cultural compromises and historical conditionings that stand below the surface of the rational stances which characterize this sort of philosophy and are arguably responsible for its academic success. Even though Logic is generally supposed to be a basic element in our very capacity to argue and evaluate argument, the fact remains that logical theory is deeply dependent on some prelogical choices. We will argue that the cultural settings that frame the very practices of philosophical reasoning and speculation should be taken seriously into account in the judgment of standards of rationality, and therefore also in the (philosophical) choice of logical theory.

Our approach, which will support a radical critique of dominant trends in philosophical logic, is both culture-sensitive and nonrelativistic, and takes inspiration from Alasdair MacIntyre's criticism of modern moral rationality. For MacIntyre, the abandonment of the complementary and mutually reinforcing features of the metaphysical grounds of traditional virtue ethics and the conception of philosophy as a goal-oriented social practice with shared standards of objective achievements embedded in a tradition of enquiry as originative of the vocabulary of moral philosophy and the uses it was put

into, entailed a detachment of such uses from the frameworks of rationality that were their proper context. Consequently, the attempts to provide the notion of a rationally grounded notion of morality (with universal scope) were invested with a degree of arbitrariness that rendered them vulnerable to the acid criticisms issued by Nietzsche and generated the apparently insurmountable difficulties faced by the proponents of opposed viewpoints in contemporary moral philosophy, unable to find a set of common standards whereby to judge of their respective commitments. MacIntyre himself extends this kind of account to describe the situation in theoretical philosophy (although only in sketch). Indeed, he establishes a deep connection between the recognition of final ends in human life and that of first principles in philosophy (not to be confounded with epistemological first principles of the Cartesian sort). In both kinds of enquiry (and MacIntyre believes that the same applies to scientific enquiries as well), the claims of philosophy to substantive and true knowledge could be vindicated through the capacity of a "research program" not only to solve its own internal conceptual and technical problems (as tends to be the case with most fashionable modes of philosophizing) in a productive, non-degenerative manner, but also to provide a coherent account of its own situation as a goal-oriented practice, as well as an account of the rivals' inadequacies by their own standards and also to construe, in a manner that articulates properly the former features, a narrative able to integrate the reasons of its own successes and the causes and consequences of its rivals' failure. Moreover, for MacIntyre, the understanding of rational enquiry both influences and suffers the influence of the cultural environment and the institutionalized modes of life and the social organization of knowledge that prevail in a given society, so that it can either foster or hinder the social importance of philosophy itself.

We believe that the philosophical uses of modern logic show signs of a situation similar to that ascribed by MacIntyre to modern practical rationality. The rejection of the Aristotelian/Scholastic tradition that worked the meanings of the logical vocabulary and the logical principles ended up in depriving such resources of the context that gave them sense and coherence – a context deeply dependent on some substantial assumptions about human nature and the aims of rational enquiry. Nowadays, a principle like noncontradiction or the excluded middle can be reduced to a formula that may be indifferently assumed as an axiom or derived as a theorem within a formal system – or simply abandoned as a thesis within some rival one. The substitution of semantics for metaphysics and the common appeal to "linguistic uses" to establish the normative patterns of reasoning fares no better than modern philosophy's typical appeal to alleged intrinsic constraints of the human mental makeup, given the very multiplicity of irreducible and incompatible alternative "reports". Quine, a major defender of the sovereignty of classical logic relies on fundamentally pragmatic grounds, and allows for the revision of logic as much as he does for any other part of the scientific enterprise. Tarski, one of the constructors of the orthodoxy, in dealing with the concept of logical consequence, admits his own account to be an abstraction from "ordinary use" inescapably tainted with a mark of arbitrariness. This floating margin was to become the starting point for Beall's and Restall's defense of logical pluralism. In either case, we see a subordination of logic to foreign and mutable interests, so that philosophical reason finds no room for the claim to a kind of knowledge at once real, substantive and both socially and existentially relevant.

4.23 Temporal Passage and the Logic of Absolute Becoming

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Abstract. According to an intuitive account, the passage of time consists in the continuous becoming past of previously present states of affairs. Accordingly, a necessary condition for passage being an objective (viz. mind independent) feature of reality, is that tense properties (i.e. presentness, pastness and futurity) be conceived as objective features of reality too. Conceiving of tense properties as objective, however, as I shall argue, is far from being a sufficient condition for passage itself. Here are three related distinctions in the philosophy of time which bear on the issue of passage. First, there is the distinction between (1) those who believe that things change only if reality itself changes, i.e. only if the totality of monadic states of affairs that exists or obtains at a time is different from that which exists or obtains at other times (also known as Dynamicists); and (2) those who think that what states of affairs constitute reality is not something that depends on what time it is, or on any other temporal perspective (the Staticists). Sure things can have different properties at different times, even under a Static conception of the world; but whether a thing instantiates a property (at a given time) or not, under a Static account, is not itself a matter that “changes” over time in any sense. Another major divide is that between those who believe that tense predicates, e.g. Present, Past or Future, refer to mind independent properties or aspects of reality, and those who don’t. The former thesis is often referred to as the A-theory of time, or Tense Realism. What all Realist conceptions of tense have in common is the contention that tensed propositions (or utterances) do not have tenseless truth- conditions. Anti-realists, on the contrary, think that tense determinations merely reflect anthropocentric, mind-dependent or perspectival features of reality (Smart 1949, Grünbaum 1963, Williams 1951).

The third divide that I wish to discuss pertains to the ontology that underlies tense discourse. According to some authors (the Eternalists), past, present and future things and states of affairs, while possibly located at different temporal “positions”, all (tenselessly) exist on an equal footing. According to their foes (the non-Eternalists), on the contrary, the differences between past, present and future experiences reflect ontological distinctions. At the far end of the spectrum of non-Eternalists views is the doctrine of Presentism: the view that, necessarily, it is always true that only present

objects and states of affairs exist.Prima facie, one is tempted to think that there should be a neat correspondence between Staticism, Anti-realism and Eternalism (the B-package), on the one side, and Dynamicism, Realism and non-Eternalism (the A-package) on the other. Dynamicists, Realists and non-Eternalists, contrary to their respective opponents, are typically committed to the view that temporal passage is an objective feature of reality which plays an essential explanatory role in accounting for change. Indeed, it is safe to claim that the chief allure of Realist conceptions of tense is that they appear to be uniquely capable of accounting for the passage of time. Here I argue that, contrary to what is often assumed, being Realist about tense and endorsing a non-Eternalist ontology does not suffice to provide a dynamic account of change. Further, I shall argue that Realist ontologies that are purely Comparative do not have the conceptual resources to express the fact of temporal passage. Comparative A-theoretic accounts of change and passage are thereby argued to fail to deliver what they promise, i.e. to provide us with a dynamic conception of reality. The structure of the talk is as follows. In part 1 I discuss a familiar no-change objection raised against the B-package (I call it McTaggart's no-change objection). In part 2 I argue that McTaggart's no-change objection to the Static account of time can be generalized to become anobjection to the comparative nature of the account per se, rather than (merely) an objection to the tenseless ontology that it presupposes. This objection, therefore, has teeth (if it has teeth), regardless of whether the relevant comparative facts are expressed in tensed or tenseless terms. I conclude that being Realist about tense, or endorsing a non-Eternalist ontology is not enough, per se, to express the fact that we live in a dynamic world. In part 3 I introduce a largely neglected, independent no-change objection. It can be traced back to James' criticism of Russell's account of motion, and it derives from the intuition that nothing can change if it is never found in an instantaneous state of changing. In 4 I argue that the same intuition can be used to mount a no-change objection against comparative theories of change and passage in general (regardless of the ontological status of tense properties). If James' objection is a valid argument against Russell's account of motion, I shall argue, then it must also be a valid argument against comparativist accounts of passage in general. I conclude (part 5) that A-theorists must either (1) accept the conclusion that time, according to their account, does not flow, or (2) put forward an account of flow that is not comparative. Finally, I shall formalize thearguments put forward, discussing a number of recent developments in event logic which bear on the issues of passage and absolute becoming.

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4.24 A philosophical view on algebraic semantics for modal logic

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Abstract. Modal logic has been criticized with respect to its underlying ontology, mainly by Quine, the most important philosopher who developed and maintained such criticism. His main argument is founded in the view that this semantics is based on the notion of possible worlds, and this concept allows a proliferation of entities in the universe (cf. [Quine 1953]). The consequence of this is a great problem in ontological terms. However, possible-world semantics are not the only possible semantics for modal logics: there are also the algebraic semantics, for instance. The work of Jósson, Mackinsey, Tarski and Lemmon in the years forty, fifty and sixties was very important for the development of such semantics. (cf. [McKinsey 1941], [Jónsson and Tarski 1951]). Bull and Segerberg in *Handbook of Philosophical Logic* [Bull and Segerberg 1984] even claim that if the paper by Jósson and Tarski had been more widely read when it was published, the history of modal logic might have been different. The schema G^{mnpq} generalizes infinitely many modal axiom schemas (cf. [Lemmon et al. 1977]); based upon this schema, completeness results and other important properties can be proven for infinitely many systems of modal logics in purely algebraic terms. These results show that algebraic semantics is as appropriate as the possible-worlds semantics, and from the philosophical point of view algebraic semantics can thus be seen as a response to the Quinean criticisms. We have thus a philosophical justification for the study and development of both modal logic and its algebraic semantics.

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5 LFIs - 15 years of Logics of Formal Inconsistency

Date: Sep 4, 2015

This is a workshop commemorating the 15 years of the Logics of Formal Inconsistency [$\circ$]. The Logics of Formal Inconsistency (LFIs) are a family of paraconsistent logics that encompasses the majority of paraconsistent systems developed within the Brazilian tradition. In a few words, LFIs have resources to express the notion of consistency inside the object language by means of a sentential unary connective called 'ball', denoted by $\circ$, and a sentence $\circ\alpha$ is intended to express the meaning that α is consistent.

As in any other paraconsistent logic, explosion does not hold in LFIs. But the vigorous approach of the LFIs allows for the distinction between contradictions that can be accepted from those that cannot. The point of this distinction is that, no matter the nature of the contradictions a paraconsistentist is willing to accept, there are contradictions that cannot be accepted. In the LFIs, negation are explosive precisely with respect to consistent formulas:

$$\circ\alpha, \alpha, \neg\alpha \vdash \beta$$

An LFI is thus a logic that separates the sentences for which explosion holds from those for which it does not hold. The former are marked with $\circ$. For this reason, they are called gently explosive.

The idea of expressing a kind of "well-behavedness" in the object language is also found in da Costa's Cn hierarchy. In C1, the consistency of α is expressed by $\alpha\circ$, and

$$\alpha,\ \neg\alpha \nvdash \beta,\ while\ \alpha\circ\ ,\ \alpha,\ \neg\alpha\ \vdash\ \beta$$

However, in C1, the well-behavedness of a proposition α is equivalent to saying that α is non-contradictory. What the LFIs have introduced in the literature is a new way of treating consistency, an idea that reveled to be really fruitful.

Although a first step in any paraconsistent logic is the distinction between triviality and contradictoriness, the LFIs permit also the distinction between consistency and non-contradictoriness, as well as contradictoriness and non-consistency. Consistency is not more necessarily regarded as “freedom from contradiction”, but acquires an independent meaning.

We may say, pictorially, that the LFIs are the paraconsistent logics that bravely reintroduce the notion of consistency into the nonclassical picture by neatly balancing the equation:

$$contradictions + consistency = triviality$$

The idea that consistency (and inconsistency) may have an independent status in logic and be internalized by way of appropriate logical constants, autonomous from other logical constants such as negation and conjunction, led to a new revival in paraconsistency, with an untold number of applications in areas such as belief revision, formalization of contradictory belief, legal studies, knowledge representation, description logics, probability, quantum computation and linguistics. The present Workhop on the 15 years of the LFIs has the purpose of congregating logicians, philosophers and computer scientists interested in furthering of the research and in the applications of the Logics of Formal Inconsistency, and their unfoldings.

Connections to proof theory, formal semantics, foundations of set theory, model theory other logics and to traditional philosophical topics have also been developed. Topics of interest to our Workshop include, but are not limited to:

- Logical systems related to LFIs (modal, fuzzy, etc).
- The LFIs and other schools of paraconsistency
- LFIs and computer science
- Possible-translations semantics
- Non-deterministic semantics
- Semantics for LFIs in general
- Proof-theory for LFIs
- Philosophical topics on LFIs
- LFIs and linguistics
- History of paraconsistency

Keynote speakers:

- Arnon Avron, Tel Aviv University
  Talk: “What is a classical LFI, and when are two such logics identical”
- João Marcos, UFRN
  Talk: “On the birth of the LFIs: Some alternative histories”

The workshop chairs are Marcelo Coniglio and Walter Carnielli.

[○] The birth of the LFIs is here traced back to the II WCP, to the publication of the first paper mentioning "formal inconsistency" in its title, and to the earliest versions of the landmark survey paper containing a proper definition of a Logic of Formal Inconsistency. A later fundamental paper on the topic may be found here.

5.1 On a four-valued LFI born from algebra

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Abstract. The variety of tetravalent modal algebras (**TMA**) was first considered by Antonio Monteiro, and mainly studied by I. Loureiro, A.V. Figallo, A. Ziliani and P. Landini. Later on, J.M. Font and M. Rius were interested in the logics arisen by the algebraic and lattice–theoretical aspects of these algebras. They introduced a sequent calculus for (using the terminology of [Bou et al. 2009]) the *preserving degrees of truth* propositional logic associated to it. Its semantics can be characterized by the matrix formed by the four–element **TMA**-algebra which generates the variety, and one of its two prime filters. This logic is called *Tetravalent Modal Logic* ($\mathcal{TML}$).

As proved in [Coniglio and Figallo 2014], the logic generated by $\mathcal{TML}$ is paraconsistent, while (using the terminology of [Bou et al. 2009] once again) the *truth-preserving* propositional logic associated to **TMA**-algebras, where 1 is the only designated truth-value, is explosive. By considering the contrapositive implication introduced by A. Figallo and P. Landini in [Figallo and Landini 1995], both logics of **TMA**-algebras were characterized by very natural and simple Hilbert calculi (see [Coniglio and Figallo 2014]). The main feature of the contrapositive implication is that it internalizes the consequence relation (whenever just one premise is considered). In algebraic terms, it internalizes the partial order of **TMA**s. Another important aspect of the contrapositive implication is that all the operations of the **TMA**s can be defined in terms of it and the first element 0.

In this talk, we retake the question of studying the logical aspects of the logic $\mathcal{TML}$, by introducing a cut-free hypersequent calculus and a tableau system which are sound and complete for the semantics preserving degrees of truth of **TMA**-algebras. The paraconsistent aspects of $\mathcal{TML}$ are also analyzed under the point of view of *Logics of Formal Inconsistency* (**LFI**s, see [Carnielli and Marcos 2002] and [Carnielli et al. 2007]), by showing $\mathcal{TML}$ is an **LFI** gently explosive with respect to single formula.

The concept of inconsistency (or non-consistency), on the one hand, and contradiction, on the other, can be separated in the logic $\mathcal{TML}$. This is a valuable feature, from a conceptual perspective, in the universe of **LFI**s, just

satisfied by **mbC**, the weaker logic of the hierarchy presented in [Carnielli et al. 2007]. Despite the fact that it is not a functionally complete logic, it enjoys a great expressive power. For instance, it is possible to speak about the "classical" truth-values (0 and 1), as well as to identify the "non-classical" ones, namely N and B.

We shall see that it is possible define a consistency operator $\circ\alpha$ and a inconsistency operator $\bullet\alpha$ in $\mathcal{TML}$ in terms of the others, which have properties of propagation similar (in the case of $\circ$) to those of the consistency operator proposed by da Costa.

As usual with **LFIs**, it is interesting to analyze the possibility of reproducing classical propositional logic (**CPL**) inside $\mathcal{TML}$. In this sense, we present a *Derivability Adjustment Theorem* (DAT) with respect to **CPL**.

Concerning the proof theory of $\mathcal{TML}$, we show that the sequent calculus presented by Font and Rius in [Font and Rius 2000] for $\mathcal{TML}$ does not admit the cut−elimination property. So, we formulate a hypersequent calculus sound and complete with respect to $\mathcal{TML}$ which does admit this property.

Besides, by adapting the general techniques introduced in [Caleiro et al. 2005], we define a decidable tableau system adequate to check validity in the logic $\mathcal{TML}$. This system, together with the hypersequent calculus mentioned above, constitute, to the best of our knowledge, the first (proof-theoretic) decision procedure introduced in the literature for checking validity in the variety of tetravalent modal algebras, besides the four-valued truth-tables of $\mathcal{TML}$.

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5.2 Paraconsistent probability structures over LFIs

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(joint work with Juliana Bueno-Soler[2])

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Abstract. Although it is well recognized that there are two main competing schools of probability (cf. [Chuaqui 1977]) which lead to different methods of statistical inference and estimation: the frequentist (based on the laws of large numbers) and the Bayesian methods (based on Bayes' theorem), there is also a second competition, represented by the theories of logical probability (Keynes/Carnap) versus measure-theoretical approaches to probability (Kolmogorov). We intend to show that notions of paraconsistent probability, as sketched in [Priest 2006], can be very naturally defined over LFIs (cf. [Bueno-Soler and Carnielli 2015]) with interesting pragmatic features, and how Bayesian methods (expressing belief degrees) can be successfully adapted to them. We also discuss the problems of extending theories of logical probability and Kolmogorovean probability spaces to LFIs and their interconnection, as well as the question of extending possibility and necessity functions (as e.g. in [Besnard and Lang 1994]) over LFIs.

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5.3 Paraconsistency as evidence preservation: a natural deduction approach

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(joint work with Walter Carnielli)

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Abstract. The acceptance of a pair of contradictory sentences A and $\neg A$ in paraconsistent logics may be understood as the occurrence of conflicting evidence about the truth value of A. Evidence that A is true (or false), in its turn, may be understood as reasons for believing that A is true (or false). From this point of view, the notion of preservation of evidence presents itself as a topic to be further developed in paraconsistency. In *BHK* interpretation for intuitionistic logic, natural deduction rules preserve of (some sense of) construction. Analogously, we present a natural deduction sentential system designed to express preservation of (some sense of) evidence. The system is paraconsistent and paracomplete, since neither explosion nor excluded middle hold, although double negation equivalence holds. The inference rules for disjunctions, conjunctions and conditionals are obtained in two steps. First, we ask about the sufficient conditions for having evidence that a given proposition is true. Then, we ask what would be sufficient conditions for having evidence that a given proposition is false. Each step produce rules whose conclusions are disjunctions, conjunctions, conditionals and negations of these formulas. Once the introduction rules are obtained, we get the elimination rules, as suggested by Gentzen, as 'consequences' of the introduction rules. Although the system so obtained is able to express the notion of preservation of evidence, and not preservation of truth, by applying the resources of the logics of formal inconsistency, classical logic is recovered with respect to propositions whose truth value has already been conclusively established. Once classical logic is recovered, the system turns out to be able to give also an account of preservation of truth.

5.4 From Fidel structures to swap structures for LFIs

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Abstract. In 1977, M. Fidel and D. Vakarelov independently proposed a semantics for D. Nelson's logic in terms of a novel class of algebraic structures now called *twist structures*. Also in 1977, Fidel obtained for the first time the decidability of da Costa's paraconsistent systems C_n by using certain algebraic-relational semantical structures now called *Fidel structures*. Logics in the hierarchy C_n, as well as several others within the hierarchy of paraconsistent logics known as *Logics of Formal Inconsistency* (**LFIs**), cannot be characterized by a single finite matrix. Moreover, they are non-selfextensional and do not have non-trivial logical congruences, and so they bravely resist to the semantical analysis based on standard tools, like categorial or algebraic semantics. This is why the development of alternative semantical techniques for this kind of **LFIs** is deemed necessary. In this talk we introduce a new class of semantics for these **LFIs** in terms of Fidel structures, as well as a matrix semantics in terms of a derived class of twist-like structures called *swap structures*, which are basically families of non-deterministic matrices (in the sense of A. Avron and I. Lev) defined over Boolean algebras. The equivalence between both semantics is proved directly. Additionally, a proof of the decidability of these systems in terms of swap structures defined over the 2-element Boolean algebra is also obtained.

5.5 A semi-natural conditional for expressing transparent truth and consistency

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Abstract. LFIs are systems designed to express the meta-theoretical notion of consistency in the object language of a paraconsistent logic. An open challenge is to extend that expressive resources of these logics to deal with transparent truth, avoiding semantic paradoxes. This challenge is not easily accomplished since most of the LFIs have conditionals whose rules and principles are known to generate paradoxes in presence of a transparent truth predicate.

We will present an LFI called **MST** −−for "Matrix with a Semi-natural conditional with truth", where the conditions for being a "semi-natural conditional" are the ones specified by Tomova in [Tomova 2012]. The salient characteristic of this conditional is that when $v(\phi) = \frac{1}{2}$ and $v(\psi) = 0$, in **MST** the value of the conditional is $\frac{1}{2}$. This also has important consequences in the way biconditionals are treated, and those consequences help **MST** to deal with self-referential sentences. In particular, with biconditionals that can be read as expressing (in the language) "The Liar" or a "Curry sentence". But **MST** matrix is non-monotonic, and this makes harder finding a fixed-point interpretation of the truth predicate, and thus proving a non-triviality proof for the theory. In order to reach this goal, we will use a three-side disjunctive sequent calculus, named **LST**. **LST** "translates" **MST** into a disjunctive sequent language, because it can be proved that $\Gamma \vDash_{\mathbf{MST}} \Delta$ if and only if $[\Gamma|\Delta|\Delta]_{\mathbf{LST}}$ is provable.

Moreover, we will show that **LST** is non-trivial. That will help us to show, via the completeness proof of LST that **MST** is also non-trivial. This proof will involve, as usual, a cut-elimination proof for **LST**. Finally, we present some reflections and comparisons regarding our conditional, **MPT**'s conditional (see [Coniglio and Silvestrini 2014]) and Priest's conditional from **LP**.

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5.6 Dual-Valuation Logics and a New LFI

Ben Martin

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Abstract. In this paper I introduce a new family of logics, dual-valuation logics, and then concentrate on one glutty member of the family, **gD-V**, which turns out to be a new *Logic of Formal Inconsistency* (**LFI**). Interesting properties of **gD-V** are highlighted, including the fact that **gD-V** contains both 'consistent truth' and 'consistent falsity' operators, and that its modal extension **gD-Vm** doesn't contain formulae of the form $\neg\Diamond(A \wedge \neg A)$ as theorems.

Dual-valuation logics are logics that conceive of the total valuation of a propositional parameter in a zero-order logic as constituted by two relations, ε^+ and ε^-, rather than one. The first relation, ε^+, is a valuation relation and is identical to the relations found in dialetheic logics with relational semantics, such as Priest's **LP**. The second relation, ε^-, found in dual-valuation logics, however, rather than symbolising the logic's valuation relation, symbolises the logic's anti-valuation relation. To understand the function of the two relations, it's helpful to consider an analogy with the extension and anti-extension of a predicate. Just as certain objects are in the extension of a predicate P, and other objects are in P's anti-extension, so truth-values are either in the valuation set of a proposition p, or its anti-valuation set. The valuation set of p is dictated by the truth-values that p has the relation ε^+ to, and the anti-valuation set of p is dictated by the truth-values that p has the relation ε^- to. As is normally the case, valuations are relations from propositional parameters to the set of truth-values true, false, and anti-valuations are similarly relations from propositional parameters to the set of truth-values.

Every member of the dual-valuation family places two restrictions upon the membership conditions of the valuation and anti-valuation sets for a proposition. For any proposition p and truth-value t: 1) Either $p\varepsilon^+t$ or $p\varepsilon^-t$, and 2) It's not the case that both $p\varepsilon^+t$ and $p\varepsilon^-t$. Thus, it's an assumption of all dual-valuation logics that the valuation and anti-valuation sets partition the set of truth-values for every propositional parameter. Consequently, these restrictions placed on the valuation and anti-valuation sets for a proposition ensure that the metatheory of all dual-valuation logics behaves consistently. Given this, it's reasonable to consider the anti-valuation relation to be communicating which truth-values are not, classically understood, members of the valuation set of a proposition. Any other restrictions which are placed upon the membership conditions of the valuation and anti-valuation sets for a proposition are dependent upon the individual logics within the dual-valuation family.

In this talk we concentrate on one member of the dual-valuation family, the glutty but non-gappy **gD-V**, which has normal semantics for conjunction and negation, that is $v(A \wedge B)$ = $min\{v(A), v(B)\}$ and $v(\neg A) = 1 - -v(A)$. In **gD-V** both the truth-values true and false can be members of a propositional parameter's valuation set, however the valuation set for a propositional parameter must be non-empty. Thus, although a propositional parameter p can have the valuation relation to both true and false, p can only have the anti-valuation relation to either true or false. Consequently, there are three permissible total valuations for a propositional parameter in **gD-V**: 1) $p\varepsilon^{+}1$ and $p\varepsilon^{-}0$; 2) $p\varepsilon^{-}0$ and $p\varepsilon^{-}1$; and 3) $p\varepsilon^{+}1$, $p\varepsilon^{-}0$, and $p\varepsilon^{-}\varnothing$.

We demonstrate that **gD-V** is an **LFI**, by being a paraconsistent logic that is able to recapture classical validity through consistency assumptions in the logic's object language, and that unlike other well-known **LFI**'s **gD-V** has a 'consistent truth' and 'consistent falsity' operator. We also show that, as one would expect from an **LFI**, **gD-V** contains an explosive, as well as an unexplosive, negation and a bottom particle, allowing one to distinguish between three distinct forms of explosion. **gD-V**'s expressive resources as an **LFI** ensure that we can introduce new formal versions of the law of non-contradiction (LNC), with the aim of finding a suitable replacement for the current prevalent formalisation of the law as formulae of the form $\neg(A \wedge \neg A)$, which fail to fulfil the role expected of the LNC. We then show that a modal extension of **gD-V**, **gD-Vm**, possesses the interesting property of failing to include formulae of the form $\neg\Diamond(A \wedge \neg A)$ as theorems, even though the logic both respects the normal semantics for conjunction and negation, and adheres to an intuitive modal semantics. It's also demonstrated that **gD-Vm** blocks the implication from contradictions at worlds accessible from a world *w* to contradictions at *w* itself, the first modal logic with normal semantics for conjunction and negation, and intuitive modal semantics, to achieve this.

We end the paper by highlighting possible philosophical uses for **gD-V** and **gD-Vm**, and suggesting that another **LFI** member of the dual-valuation family, the glutty and gappy **gD-V**, will allow us to recapture both classical inferences and classical tautologies, offering the opportunity for glutty and gappy logics to possess all of the expressive power of classical logic and more.

5.7 Some remarks on games, normativity and contradictions

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Abstract. In *Grundgesetze* II, Frege (1903) incidentally uses the notion "conflict of rules" ("Widerstreit der Regeln") to explain what contradictions are, when he is critically evaluating some formalist accounts of mathematical practices and entities. In 1930, when Wittgenstein was preparing Waismann for representing him in a very influential panel on the Philosophy of Mathematics to be held in Königsberg, he explicitly borrows from Frege's discussions this notion, "conflict of rules", to criticize Hilbert's metamathematical enterprise, especially his account of consistency (*Widerspruchsfreiheit*). Due to these discussions with members of the Vienna Circle (1929-1932), some authors suggest that Wittgenstein could be held as a forerunner of paraconsistent logics. Indeed, Wittgenstein, during these discussions, and in other texts from the same period, reacts very tolerantly to some non-classical reasoning, especially in the presence of formal contradictions. In the beginning of the 30's, Wittgenstein's focus was neither on formal trivialization nor on any mandatory collapse of calculi which entail contradictions, but rather he was already sketching a very comprehensive anthropological account of logic. This account may help us to articulate, through the notion of normativity and rules, the nature of formal systems and the relevance of human practices in the construction of paraconsistent logics.

A very important point, which is often overlooked by some authors, is that Wittgenstein at the beginning of 1930's *did not* have the concept of language games yet, which is the favorite theoretical candidate to explain why the author of the *Tractatus* turned out to be so "tolerant" in respect to non-classical reasoning. Much before *Remarks on Foundations of Mathematics, Lectures on Foundations of Mathematics*, and *Philosophical Investigations* itself, Wittgenstein already uses some of his "mature" arguments against classical logic and realist approaches to mathematical practices. We may speculate some reasons for this turn.

In the beginning of 1930's, Wittgenstein begins to take seriously the notion of games as an illuminating metaphor for understanding Logic and Mathematics. In this sense, his latter "language games" could be more adequately

taken as a philosophical development, or better a radicalization, of some insightful consequences of holding logic as a game. For example, *Verbot* and *Erlaubnis*, which usually mark deontic aspects of actions, are not, in a philosophically relevant sense, in the world independent of individuals engaged to public practices in a community.

Holding this crucial claim as true, it is easy to see some plausibility in Wittgenstein's much controversial accounts: just as measurement systems are not in the world, that is, "real" and independent of human practices, logic should be not "out there" either. Just as any scale or system of coordinate does not have to represent something in order to be understood and applied, logical connectives do not have to stand for some metaphysical entities for being objective. In analogue to measurement systems, logic's objectivity must neither be justified by any independent reality nor by the structure of our perception nor by any deep grammar of our mind; rather, it could be justified by the stability of our practices and tacit agreements. Both, scales and logical connectives, have to be stipulated, established by our practices. If logic could be reducible to a normative sphere of instructions for practices, and Wittgenstein's thinking on games is in fact a very good candidate to make this proposal more feasible, as Lorenzen and Hintikka later on also noted, then logic could not be independent of the world of human communities and practices.

The introduction of games to understand logical systems represents an anthropological turn in the discussion of the nature of mathematics and logic, one that could bring Wittgenstein to the center of contemporary discussions on logic and the philosophy of mathematics. As a consequence of this account, we could then understand Wittgenstein's remarks on contradictions in systems in the beginning of the 30's: what should *we do* when we have contradictory rules in a system, if rules should be taken as instructions to action or norms of some rule-governed practices? "Nothing!" This is exactly Wittgenstein's answer to discovered contradictions: We do not have to fear trivialization, but inaction, or better, the paralysis of our activities.

In this contribution, we will not engage in the evaluation of Wittgenstein being a real forerunner for some non-explosive logics, but rather we will investigate why and how the notion of rules in a game could be a seminal philosophical alternative in understanding the nature of contradictions without the appeal to *dialetheias*.

6 TRS Reasoning School

Dates: Aug 31-Sep 4, 2015

(with a bonus tutorial on Aug 24-28, 2015, jointly with the Workshop DIMAp 30 Anos)

Our School was designed to have something of interest to offer to each and every participant logician (or logician-to-be)! This means that each participant is bound to find something of interest, and every participant is free to show interest in something!

Here are the distinguished **tutorialists** of our event:

- Benjamín Bedregal, UFRN
- Björn Lellmann, TU-Wien
- Carlos Prolo, UFRN
- Cassiano Terra Rodrigues, PUC-SP
- Christian Strasser, Ruhr-Universität Bochum
- Cláudia Nalon, UnB
- David Déharbe, UFRN
- Edward Hermann Haeusler, PUC-Rio
- Frank Sautter, UFSM
- Ivan Varzinczak, UFRJ
- Jonas Becker Arenhart, UFSC
- Marcelo Finger, USP
- Marcos Silva, UFC
- Mauricio Ayala-Rincón, UnB
- Regivan Santiago, UFRN
- Renata de Freitas, UFF
- Revantha Ramanayake, TU-Wien
- Walter Carnielli, UNICAMP

Note: TRS is an acronym for “TRS Reasoning School”.

6.1 Approximate reasoning

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Federal University of Rio Grande do Norte

Abstract. The human reasoning has, in general, an approximate nature. For example, if you state: "If the night is very cold, then the majority of my guests will not attend my party". This sentence considers imprecise aspects and is not appropriately captured in a standard precise reasoning, in which thresholds must be fixed beforehand; e.g. below 10 Degree Celsius establishes that "Night is very cold" and the percentage which represents the "majority of my guests" is 51 %.

Logic can be understood as the science or discipline which studies the formal principles of reasoning. In particular, Fuzzy Logic provides a precise framework to deal with such approximate reasonings. For us, approximate reasoning here means the process from which imprecise conclusions are deduced from imprecise premises. This deductive process is based on Fuzzy Logic, in which predicates are declared through the do called Linguistic Variables, the evaluation of sentences considers imprecise truth tables and the inference rules are graded.

In this tutorial we provide an overview of such framework.

6.2 Reasoning with natural language

Carlos Prolo

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Abstract. "All men are mortal. Socrates is a man. Therefore, Socrates is mortal." This classical example of syllogism, for many attributed to Aristotle, has been used by many authors to illustrate how formal logical systems may be used to model natural language communication. In this tutorial, although we will indeed acknowledge that they constitute a fundamental basis to understanding how human language work, we will do it in a very critical, contrarian-like way, showing how logical inference systems break easily when naively applied to natural language reasoning.

In human dialogues, participants conventionally do not follow straightforwardly the inference models that one could naively try to impose on them. However, strangely enough, they are still aware of these models to different extents. An interesting intertwining between inferential logic, lexical contents, common sense, world knowledge, and other not so clearly understood aspects of language, which govern human communication, are also used in jokes, and give rise to fallacious arguments, misunderstandings, disputes, arbitration and, of course, funny robot talking in movies.

The main purpose of this tutorial is to arouse in the students a taste for the challenges waiting to be tackled if we want some day to do automated processing of natural language in a minimally interesting way

6.3 Pragmatic reasoning

Cassiano Terra Rodrigues

PUC-SP

Abstract. The term "pragmatism" is usually held to mean an attitude completely devoid of theoretical concerns, thoroughly directed to sheer practical results. The very expression "pragmatic reasoning" would seem to be a contradiction from that perspective. Though common and widespread, it is a poor oversimplification. In order to understand why, this tutorial will deal with C.S. Peirce's definition of pragmatic and pragmatism. Peirce is commonly regarded as the "father of pragmatism", although he distanced himself from W. James and other self-proclaimed pragmatists. Peirce claimed his pragmatism derived from his readings of Kant, combined with his experimental laboratory training as a chemist and as an astronomer. In fact, his scientific training will allow him to abandon the subjectivist and idealistic tones from modern philosophy, and to make the sharp Kantian distinction between praktisch and pragmatisch not so sharp, thus blurring the boundaries between theoretical and practical reasoning. In this way, his treatment of abduction as a form of logically weak though heuristically strong reasoning, as well as his distinction between theorematic and corollarial kinds of deduction constitute the very core of his definition of pragmatism (or pragmaticism, as he at a certain point preferred) as the logic of discovery, and a form of methodeutics (not merely a method, but the method of methods). The topics to be presented in the tutorial are: a) Peirce's (earliest) criticism of psychologism in logic; b) Logic and pragmatism in the context of his classification of the sciences of discovery; c) From kinds of reasoning to the method of science; d) Pragmatic reasoning and the possibility of metaphysics.

6.4 Non-monotonic reasoning

Christian Strasser

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Abstract. Defeasible reasoning is indispensable when dealing with a world full of uncertainties: we constantly draw conclusions that we may reject later in view of new information. For instance, when noticing that the streets are wet, I infer that it has been raining. However, once I discern that the roofs are not wet, I retract my previous inference. In situations like this, we make inferences from premises that do not warrant that our conclusions holds: they only warrant that the a conclusion is sufficiently likely. Defeasible reasoning is not restricted to everyday contexts. It is also abundant in the (pure and applied) sciences and in expert reasoning. E.g., when diagnosing a patient, John, who shows signs that best fit hyperthyroidism, a physician may conclude that John should be tested further for this condition. However, as soon as our physician is informed that John's thyroid has been removed, she will retract her previous inference. As these examples indicate, defeasible reasoning comes in many forms: we reason from effect to cause (abduction), we make generalizations (induction), we reason on the basis of what is normally or typically the case (default reasoning), we infer on grounds of the information our senses give us about our environment, etc. In order to explicate and evaluate such reasoning processes, formal methods were developed: non-monotonic logics. In this tutorial we will discuss some central approaches in nonmonotonic logic.

6.5 Machine-oriented reasoning: 50 years of the resolution principle

Cláudia Nalon

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Abstract. In 1965, John A. Robinson published the paper "A Machine-Oriented Logic Based on the Resolution Principle", launching the bases for automated deduction as we now know. The resolution principle combines unification on demand together with a cut-elimination rule, building on the theoretical works by Prawitz and Herbrand and also improving on previous automated heuristics for proof search developed by Davis-Putnam. The result is an elegant, easy to implement, sound, and complete proof procedure with only one inference rule for dealing with validity in First-Order logic. In this tutorial, we celebrate the 50th anniversary of Robinson's foundational work by looking at the theory behind resolution-based systems for classical logics (propositional and first-order), the main results concerning the theoretical and practical development of such systems, and the state of the art heuristics that make of this proof procedure one of the most widely implemented and successful tools for automated reasoning.

6.6 Modal reasoning through resolution

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Abstract. Continuing the celebration of the 50th anniversary of J. A. Robinson's work on the resolution principle, in this tutorial we will examine resolution-based proof methods for propositional modal logics based on the axioms K, T, D, B, 4 and 5. We will briefly review the proof method proposed by Robinson for classical propositional logic as well as the basics of modal logics. We will discuss what needs to be taken into consideration when adapting the classical method to deal with the satisfiability problem for modal languages and look at two different resolution-based methods for families of monomodal logics: the clausal method proposed by Mints and the non-clausal destructive procedure proposed by Fitting.

6.7 Reasoning to verify computer programs

David Deharbe

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Federal University of Rio Grande do Norte

Abstract. Being able to guarantee the correctness of a computer program is of paramount importance for the development of safety-critical systems with embedded software. This course will present a formal system of reasoning known as Hoare Logic that underlies many current program verification systems. This course will be heavy on examples and light on theory.

6.8 Propositional reasoning complexity

Edward Hermann Haeusler

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Abstract. The computational complexity of SAT and TAUT for purely implicational propositional logic is known to be PSPACE-complete. Intuitionistic Propositional Logic is also known to be PSPACE-complete, while Classical Propositional Logic is CO-NP-complete for Tautology checking and NP-complete for Satisfiability checking. We show how proof-theoretical results on Natural Deduction help analysing the Purely Implicational Propositional Logic complexity regarded its polynomial relationship to Intuitionistic and Classical Propositional Logics. The main feature in this analysis is the subformula property. We extended the polynomial reduction of purely implicational logic to any propositional logic satisfying the subformula property. We examine PROPOSITION I: Any propositional logic satisfying the subformula principle is in PSPACE.

Using Proposition I, we: (1) Conclude that some well-known logics, such as Propositional Dynamic Logic and S5C (n = 2), for example, hardly satisfy the subformula property. On the other hand, these logics have proof procedures. They are seen to be mechanizable despite being EXPTIME-complete; (2) Show that finitely many-valued propositional logics are in PSPACE, and when they include the minimal implicational logic they are PSPACE-complete.

We point out some facts and discuss some questions on how the subformula property is related to the mechanization of theorem proving for propositional logics. The relationship between feasible interpolants and the subformula property is discussed. Some examples remind us that the relationship between normalization of proofs and the subformula property is weak. Finally we discuss some alternative criteria to consider a logic to be mechanizable.

The tutorial starts by a brief review of computational complexity.

6.9 Dynamic propositional reasoning

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Abstract. Three complementary approaches to formal logic can be distinguished: a static one, a kinematic one, and a dynamic one. The static approach investigates arguments in isolation; the kinematic approach investigates them in the framework of chains of arguments; and the dynamic approach investigates them in the framework of the interplay between the debaters. In this tutorial I will examine Classical Propositional Logic from a dynamic perspective. I will use extensively normal forms, and the notion of information will be used as the fundamental semantic notion. The following topics will be examined: 1) Static, kinematic, and dynamic approaches to formal logic; 2) Truth versus information; 3) Normal forms for Classical Propositional Logic; 4) Informational validity, strengthening, and weakening; 5) Types of opposition; 6) Enthymematic arguments; 7) Lush arguments; 8) Philosophy of logic behind the adopted approach.

6.10 Reasoning with description logics

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Abstract. This tutorial is an introduction to Description Logics in the context of knowledge representation and reasoning. Description Logics (DLs) are a family of logic-based knowledge representation formalisms with interesting computational properties and a variety of applications. In particular, DLs are well-suited for representing and reasoning about terminological knowledge and constitute the formal foundations of semantic web ontologies. There are many different flavors of description logics with specific expressiveness and applications, an example of which is ALC and on which we shall focus in this tutorial. The outline of the tutorial is as follows: We start with an introduction to the area of Knowledge Representation and Reasoning (KRR) and the need for representing and reasoning with terminological knowledge, which stands as the main motivation behind the development of DLs. We then present the description logic ALC, its syntax, semantics, logical properties and proof methods. Finally we illustrate the usefulness of DLs with the popular Protégé ontology editor, a tool allowing for both the design of DL-based ontologies and the ability to perform reasoning tasks with them.

6.11 Non-reflexive logics in quantum reasoning

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Abstract. Quantum mechanics is widely regarded as presenting many challenges to orthodox forms of reasoning. Among some of those challenges, quantum theory is not only said to violate some form of the law of distribution in classical propositional logic but, also, according to some philosophers and popular accounts of the theory, to involve contradictions in some circumstances (for instance, in the case of Schrödinger's cat, which is said to be dead and alive at the same time). In this tutorial, we shall focus in challenges presented by quantum theory to the concept of identity. Our first step is to present the main arguments advanced to the effect that quantum entities seem to violate traditional laws of identity as they are formalized in classical first-order logic with identity. According to an interpretation of the theory (which we shall discuss), quantum entities do not behave in complete agreement to those laws, and are typically called non-individuals. We shall also present a family of logics designed to deal with non-individuals, called non-reflexive logics. The most well-known system of such logics is quasi-set theory, with which we shall be mainly concerned. As a third goal, we discuss some difficulties advanced to the non-reflexive approach to quantum reasoning. The difficulties come from both logical as well as metaphysical perspectives.

6.12 Logic-probabilistic reasoning

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Abstract. We present the intersection of Logical Reasoning and Probabilistic Reasoning, without any assumption of probabilistic independence. We present probabilistic logic from first principles, and relate the probabilistic version of problems with their classical version. We relate classical satisfiability (SAT) with probabilistic satisfiability (PSAT), and analyze how to bridge between these two forms of reasoning. We also discuss the interesting phenomenon: how come SAT and PSAT are both NP-complete if the latter is, or seems, so much more complicated than the former?

In the second part,we discuss issues related to Logic Probabilistic modelling. We discuss how to compute a set of candidate solutions by minimizing an inconsistency measure between an initial distribution and a set of logic-probabilistic restrictions. Once we have a consistent distribution, we discuss on the Maximum a Posteriori (MAP) method of choosing "the" solution. We also discuss on the problems of defining measures of inconsistencies in probabilistic theories, and show its differences from measuring inconsistency in non-probabilistic theories. We finalize by discussing some applications.

6.13 Reasoning with colors

Marcos Silva

UFCE

Abstract. Colours can be found in many central problems in the History of Philosophy: from Aristotle's examples for exclusions by contrariety and problems for the Principle of Excluded Middle to the collapse of Wittgenstein's early Philosophy; from the discussion on the nature of secondary qualities in Modern Philosophy to more recent puzzles in the Hard Problem of Consciousness; from natural candidates for the synthetic a priori to the refusal of a sharp distinction between shape and content in Aesthetics. In this tutorial we will focus on some issues in History and Philosophy of Logic concerning the conceptual organization of colours. First, we will examine how some reasoning with colours has dramatically challenged Wittgenstein's influential Tractarian account of logic and how the so-called Colour Exclusion Problem has conducted his thoughts to a strong conventionalism about logical rules. Second, still regarding colour systems, we will explore some further questions about two philosophically relevant distinctions: i) between formal and material incompatibility and ii) between contradiction and contrariety.

6.14 Formal reasoning with PVS

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Abstract. Among a great variety of proof assistants PVS has been chosen for this tutorial because its prover interface allows development of formal proofs that are very close to analytical proofs one can find in textbooks and papers. The objective of this tutorial is provide computer science, math and logic students and researchers with a general view about how reasoning techniques can be applied to formalize properties (presented as mathematical lemmas and theorems) related with the correct specification of computational objects (software and hardware). We hope this tutorial, on the one side, will bring a useful first view to undergraduate and graduate students about the use of proof assistants, in general, and, on the other side, an instrumental view of the application of PVS reasoning techniques, in particular, to researchers in formal methods.

6.15 Diagrammatic reasoning

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Abstract. Until the last century, with one possible notable exception, diagrams were considered as just an aid to discovering and to communicating mathematical theorems. Drawings could be used to think about math, but a proper proof of a theorem would be a text, a sequence of sentences. Usual logical systems use sentences (formulas) to represent information and inference rules are defined to "manipulate" such sentences. In the last two decades, however, various logical systems that use diagrams to represent information (instead of sentences) and, consequently, whose inference rules are defined to obtain diagrams from sets of diagrams, are designed and developed formally. The first soundness and completeness proofs for such a system, based on Venn diagrams, are due to Sun-Joo Shin (1995). In this tutorial I will present some examples of diagrammatic logics.

6.16 Proof-theoretical reasoning (part 1 and part 2)

Revantha Ramanayake and Björn Lellmann
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Abstract. Logic is concerned with the study and use of valid reasoning. The most well-known logics are classical propositional and first-order logic. Nevertheless, various other forms of reasoning are needed to model the different applications and situations that arise in practice, thus giving rise to many new logics distinct from classical logic. The study of these logics provides an understanding of the underlying applications and also leads to new solutions e.g. automated support. Important theoretical questions in this context include whether there is a procedure to decide if a given statement represents a valid inference in the logic or not (decidability), if decidable, then what is the optimal complexity of a decision procedure, the construction of focussing methods for effective proof-search, and questions regarding further meta-theoretic properties of the logic such as consistency or interpolation. The basis for the proof-theoretical approach to reasoning is the notion of proof in the logic, and in particular, the study of formal proof systems of the logic. A major early achievement is Gentzen's introduction of the sequent calculus as a formal tool for studying the structure of proofs in classical and intuitionistic logic. The main result is the cut-elimination theorem which shows that proofs can be reduced to a normal form where the statement to be proved is constructed systematically from smaller statements. Gentzen used this result to give a proof of consistency of Peano arithmetic using a suitable induction principle. Subsequently, the sequent calculus has been adapted and used (together with a corresponding cut-elimination theorem) to prove meta-theoretical results for many modal and substructural (non-classical) logics that arise (and are applied) in diverse fields such as knowledge representation and to reason about multi-agent systems, resource-bounded programs and ethical problems.

In these tutorials, we introduce the sequent calculus and prove the fundamental cut-elimination theorem. Next we introduce normal and non-normal modal logics and substructural logics and corresponding sequent calculi and use the cut-elimination theorem to obtain meta-theoretic results such as decidability and complexity of proof search. We also consider the vexing question of how to introduce sequent calculi for new logics of interest, indicate the limitations of the sequent calculus and discuss generalisations such as hypersequent and labelled sequent systems that have been proposed in order to obtain proof-calculi with cut-elimination for larger classes of logics.

6.17 Reasoning with polynomials

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Abstract. The method for automatic theorem proving proposed in my "Polynomial ring calculus for many-valued logics" ([Carnielli 2005]) is an algebraic proof mechanism based on handling polynomials over finite fields. Although useful in general domains, as in first-order logic, certain non-truth-functional logics and even in modal logics, the method is particularly apt for deterministic and non-deterministic many-valued logics, as will be shown in this tutorial. The term "polynomizing" refers to the uses of polynomial-like representations as a reasoning strategy and as a tool for scientific heuristics. I intend to show how proof-theory and semantics for classical and several non-classical logics, as well as algebrization of logics, can be approached from this perspective, and discuss the assessment of this prospect, in particular on what concerns recovering certain ideas of George Boole in unifying logic, algebra and the differential calculus.

The method for automatic theorem proving, proposed in [Carnielli 2005] and subsequently developed in several other publications, is an algebraic proof mechanism based on handling polynomials over finite fields. Although useful in general domains, as in first-order logic, certain non-truth-functional logics and even in modal logics (see [Agudelo and Carnielli 2011]). the method is particularly apt for deterministic and non-deterministic many-valued logics, as I plan to show in this tutorial. The term "polynomizing" refers to the uses of polynomial-like representations as a reasoning strategy and as a tool for scientific heuristics. I intend to show how proof-theory and semantics for classical and several non-classical logics, as well as algebrization of logics, can be approached from this perspective, and discuss the assessment of this prospect, in particular to recover certain ideas of George Boole in unifying logic, algebra and the differential calculus.

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